LA CONTRIBUCIÓN DE A.N. KOLMOGOROV A LA TEORÍA DE LA TURBULENCIA

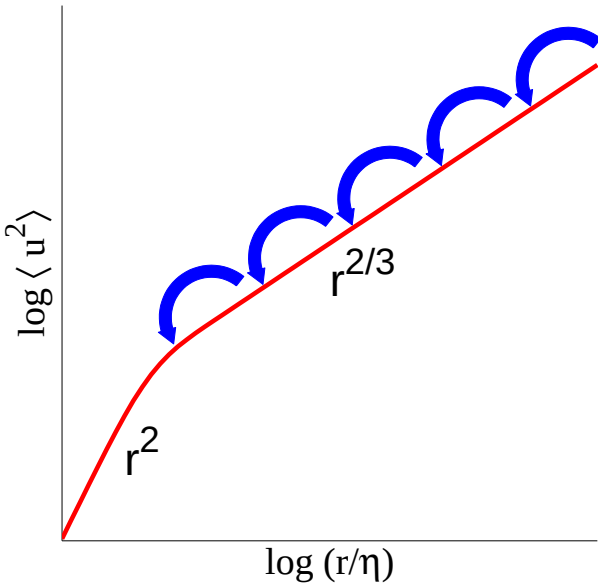
Javier Jiménez

E.T.S. Ingenieros Aeronáuticos

Universidad Politécnica Madrid

R.A.C., 4/02/2004

¿DE VERDAD QUE KOLMOGOROV FUE UN ESTADÍSTICO?



KOLMOGOROV SOBRE TURBULENCIA (1938-1950)

Kolmogorov, A.N. (1941) The local structure of turbulence in incompressible viscous fluids at very large Reynolds numbers. *Dokl. Akad. Nauk. SSSR*, 30, 299–303. (150)

Kolmogorov, A.N. (1941) On the degeneration of isotropic turbulence ... *Dokl. Akad. Nauk. SSSR*, 31, 538–541.

Kolmogorov, A.N. (1941) Dissipation of enery in isotropic turbulence. *Dokl. Akad. Nauk. SSSR*, 32, 19–21. (35)

Kolmogorov, A.N. (1942) Equations of turbulent motion ... *Izv. Akad. Nauk. SSSR ser. Fiz.*, 6, 56–58.

Kolmogorov, A.N. (1949) On the breakage of drops ... *Dokl. Akad. Nauk. SSSR*, 66, 825–828.

Kolmogorov, A.N. (1962) A refinement of previous hypotheses concerning the local structure of turbulence. *J. Fluid Mech.* 13, 82–85. (35)

LA ECUACIÓN DE LA ENERGÍA

$$\boxed{\nabla \cdot \mathbf{u} = 0}$$

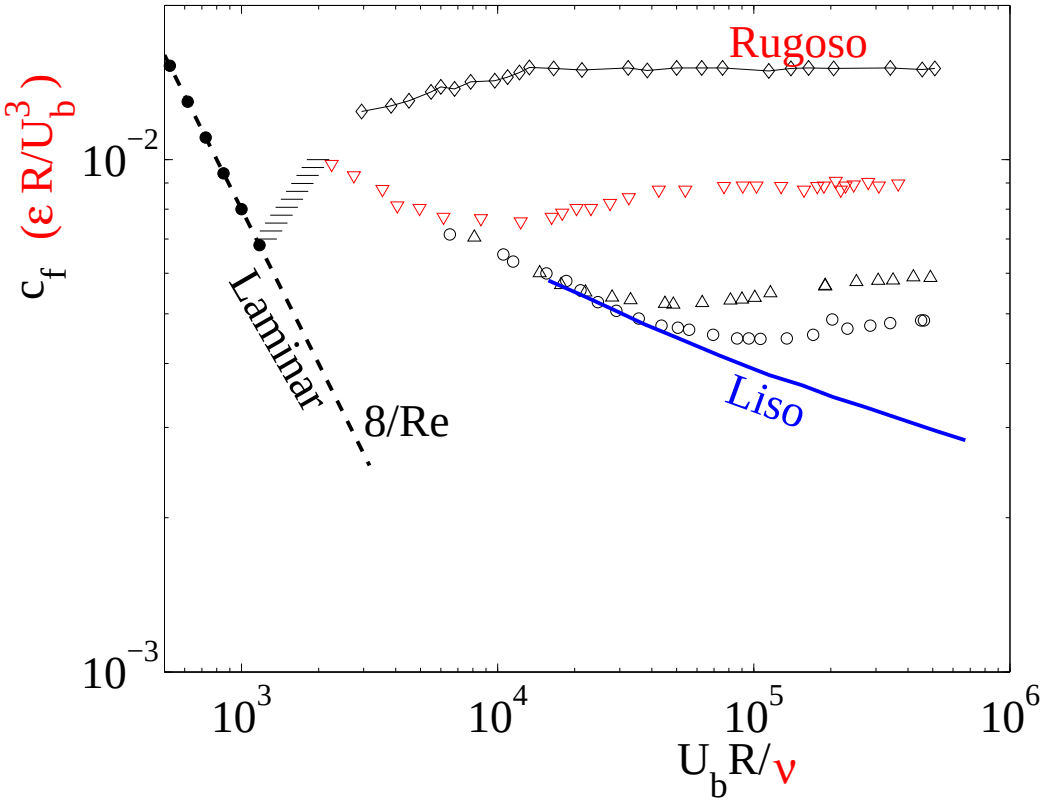
$$\mathbf{u} \bullet \quad \boxed{\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{u}}$$

Energía Cinética: $\boxed{K = \frac{1}{2}\, \mathbf{u} \cdot \mathbf{u}}$

$$\boxed{\partial_t K + \nabla \cdot \phi = -\nu |\nabla \mathbf{u}|^2}$$

<u>Disipación de Energía:</u> $\boxed{\varepsilon = \frac{\nu}{V} \int_V |\nabla \mathbf{u}|^2 \, \mathrm{d}V.}$

EL PROBLEMA DE LA TURBULENCIA



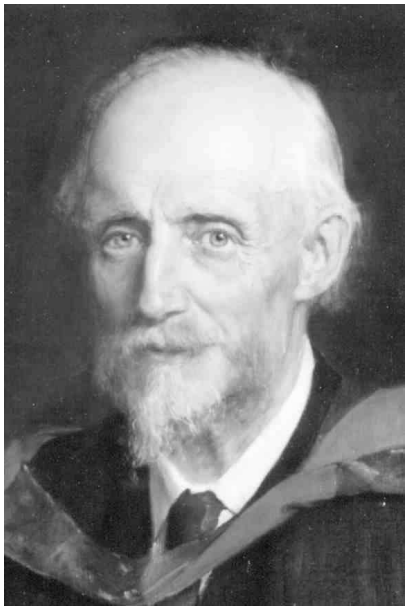
Hägen (1854)
Darcy (1857)
Boussinesq (1877)

$$\varepsilon = \nu \langle |\nabla \mathbf{u}|^2 \rangle \underset{\nu \to 0}{\longrightarrow} \text{cte.}$$

$$\boxed{\langle |\nabla \mathbf{u}|^2 \rangle \sim \nu^{-1} \underset{\nu \to 0}{\longrightarrow} \infty}$$

u es un FRACTAL

LA DESCOMPOSICIÓN DE REYNOLDS



Reynolds (1894)

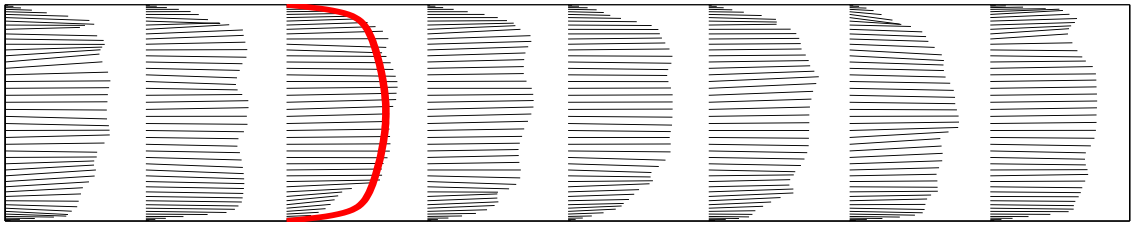
$$\boxed{\begin{aligned} u &= U(x,t) + \widetilde{u}(x,t) \\ U &= \langle u \rangle; \quad \langle \widetilde{u} \rangle = 0 \end{aligned}}$$

<u>Ecuaciones Promediadas</u>

$$\partial_t \mathrm{U} + \mathrm{U} \cdot \nabla \mathrm{U} + \frac{1}{\rho} \nabla P = -\nabla \cdot \langle \widetilde{u} \otimes \widetilde{u} \rangle + \nu \nabla^2 \mathrm{U}$$

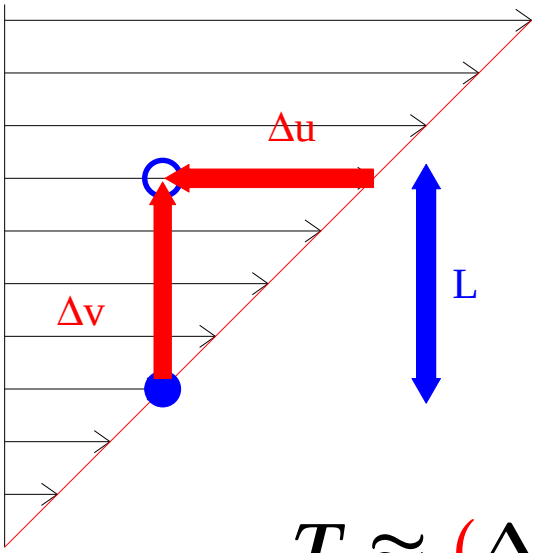
'Esfuerzos' de Reynolds: $\boxed{T = -\langle \widetilde{u} \otimes \widetilde{u} \rangle}$

$$\boxed{\varepsilon = T : \nabla U}$$



LA INFLUENCIA DE LA TEORÍA CINÉTICA DE GASES

Boussinesq (1877)
Prandtl (1925)
von Kármán (1930)
...
(Modelos para Ingeniería)



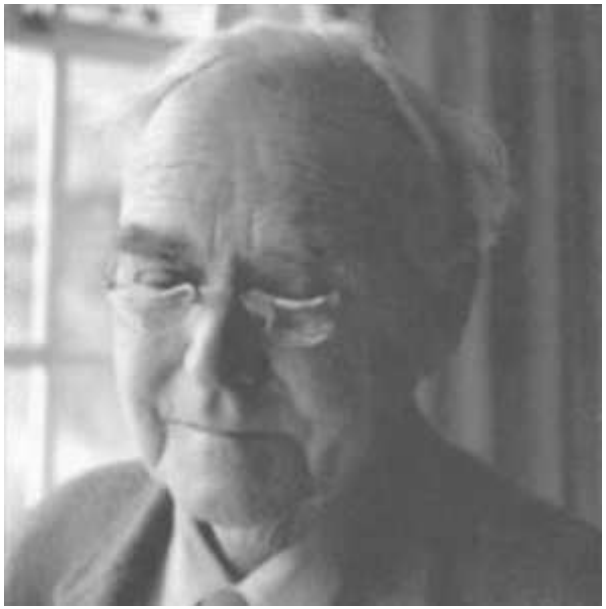
$$T = -\langle \widetilde{u} \otimes \widetilde{u} \rangle \approx -\Delta u \Delta v$$

$$T \approx (\Delta v)\,(L \partial_y U) = \nu_\varepsilon \partial_y U$$

'Viscosidad' Turbulenta:

$$\boxed{\nu_\varepsilon = \Delta v L}$$

ESTADÍSTICA DE LAS FLUCTUACIONES

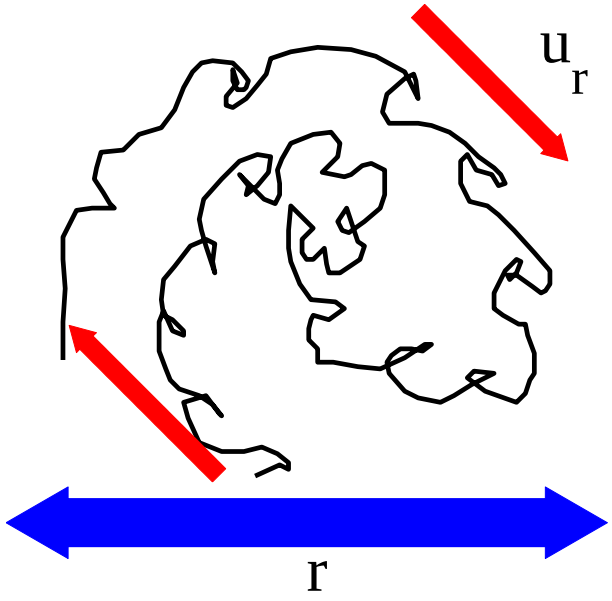


Taylor (1935)
von Kármán (1937)
Kolmogorov (1941)
Batchelor (1953)
Kraichnan (1962)
...

(Física)

Campos estocásticos
Correlaciones
Espectros
Funciones de estructura
Turbulencia Homogénea Isótropa
...

‘TORBELLINOS’



Tiempo ‘Inercial’: $$\boxed{T_I = \frac{r}{u_r}}$$

Tiempo ‘Viscoso’: $$\boxed{T_\nu = \frac{r^2}{\nu}}$$

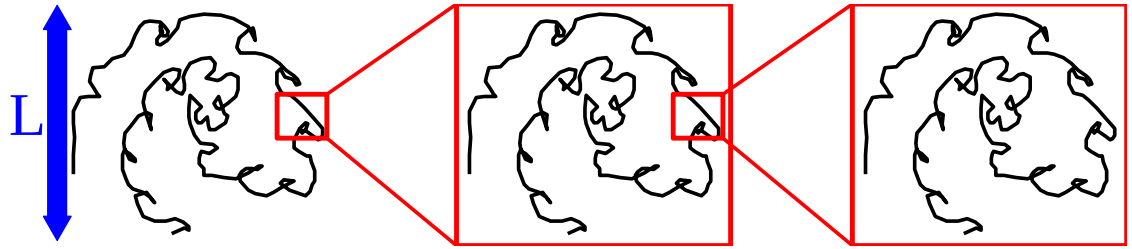
$$\boxed{\frac{T_\nu}{T_I} = \frac{r u_r}{\nu} = Re}$$

$$Re \left\{ \begin{matrix} \gg \\ \ll \end{matrix} \right\} 1 \Rightarrow T_I \left\{ \begin{matrix} \ll \\ \gg \end{matrix} \right\} T_\nu \Rightarrow \text{Viscosidad} \left\{ \begin{matrix} \text{No} \\ \text{Si} \end{matrix} \right\} \text{Importa}$$

LA CASCADA DE ENERGÍA (Kolmogorov, 1941)

$$\frac{u_r r}{\nu} \gg 1 \quad \Rightarrow \quad \text{Torbellinos No viscosos (isótropos)}$$

(Richardson, 1926)



$$r_1 \Rightarrow r_2 = \frac{r_1}{2}$$

$$E_1 = u_1^2$$
$$T_1 = \frac{r_1}{u_1}$$

$$\Rightarrow \quad \frac{\mathrm{d}E_1}{\mathrm{d}t} = \varepsilon \approx \frac{E_1}{T_1} = \frac{u_1^3}{r_1} = \frac{u_2^3}{r_2} = \ldots$$

$$u_r \sim (\varepsilon r)^{1/3}$$

LA ESCALA VISCOSA

$$\boxed{Re_r = \frac{r u_r}{\nu} \sim \frac{\varepsilon^{1/3}}{\nu} r^{4/3}}$$

Decrece con r

Escala de Kolmogorov: $Re_r = 1 \quad \Rightarrow \quad \boxed{r = \eta \approx \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}}$

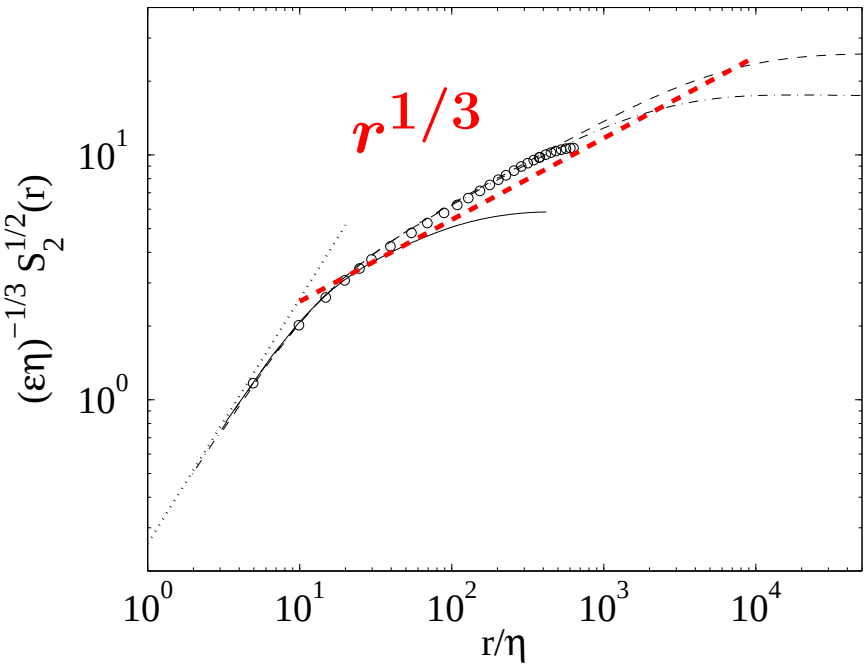
$r \ll L \quad \Rightarrow \quad \boxed{u_r = (\nu\varepsilon)^{1/4} F(r/\eta)}$

$\eta \ll r \ll L \quad \Rightarrow \quad \boxed{S_2(r) = \langle (\Delta u)^2 \rangle = c_k (\varepsilon r)^{2/3}}$

$r \approx \eta \quad \Rightarrow$ Suave: $\boxed{\nu |\nabla \mathrm{u}|^2 \approx \nu \frac{(\nu\varepsilon)^{1/2}}{\eta^2} \approx \varepsilon}$

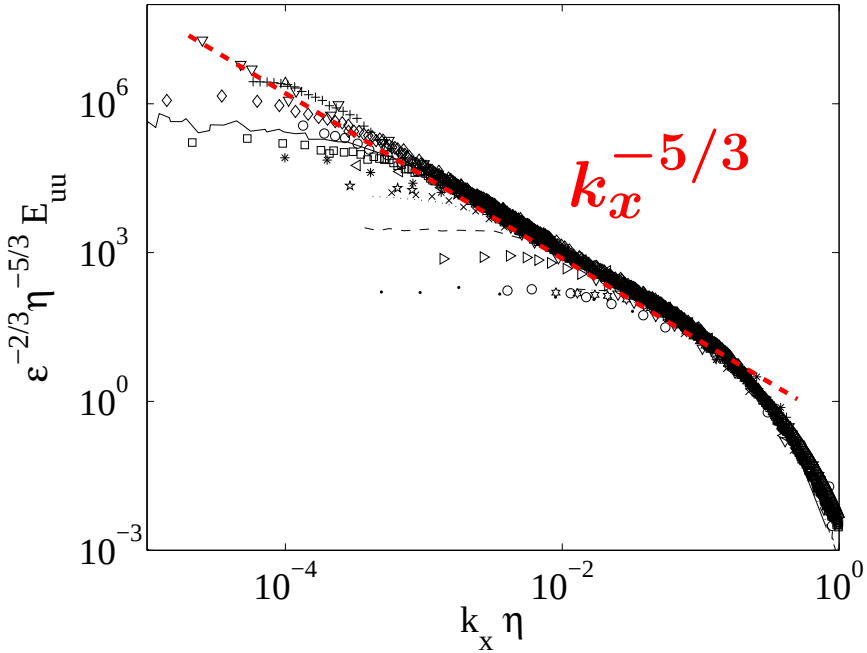
COMPROBACIÓN EXPERIMENTAL (1955−)

F. de Estructura: $S_2^{1/2}$



(Kolmogorov 1941)

Espectro de Potencia



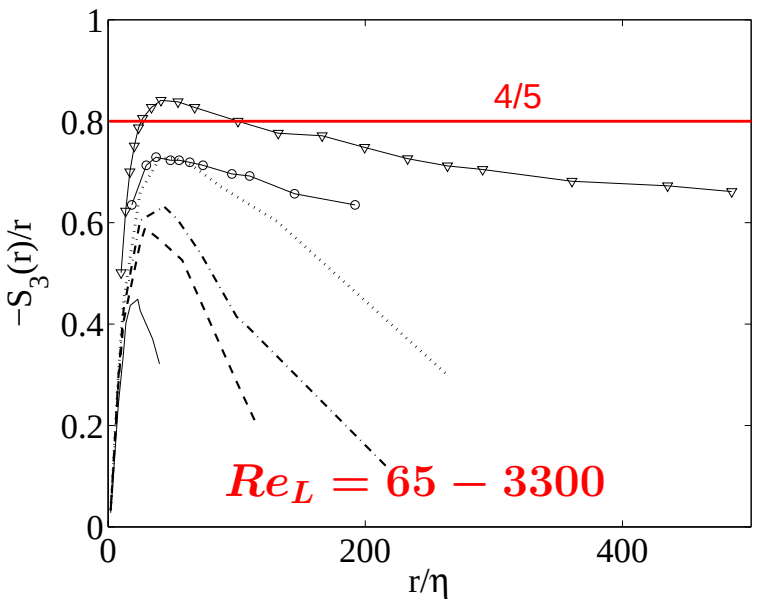
(Obukhov 1941)

ECUACIÓN 4/5 (Kolmogorov 1941c)

Energía S_2 de $\Delta u(r)$: (Kármán & Howarth, 1938)

$$\partial_t S_2 + \nabla_r \cdot \Phi_\Delta = \boxed{\begin{matrix} -\langle u \cdot F_2 \rangle \\ O(r/L)^2 \end{matrix}} + \boxed{\begin{matrix} \nu \nabla_r^2 S_2 \\ O(\eta/r)^{4/3} \end{matrix}}$$

$$\eta \ll r \ll L \;\Rightarrow\; \Phi_\Delta = 0 \;\Rightarrow\; \boxed{S_3(r) \approx -\frac{4}{5}\varepsilon r}$$



$$S_3 = \langle (\Delta u)^3 \rangle \sim \varepsilon r$$

$$\Downarrow$$

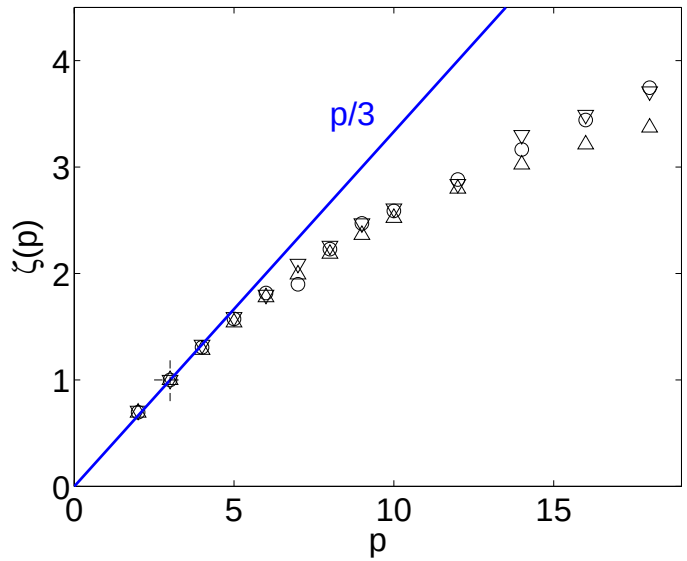
$$\boxed{S_2 = \langle (\Delta u)^2 \rangle \sim (\varepsilon r)^{2/3}}$$

INTERMITENCIA

Batchelor (1949), Kolmogorov (1962), Novikov (1970), Frisch (1975), ...

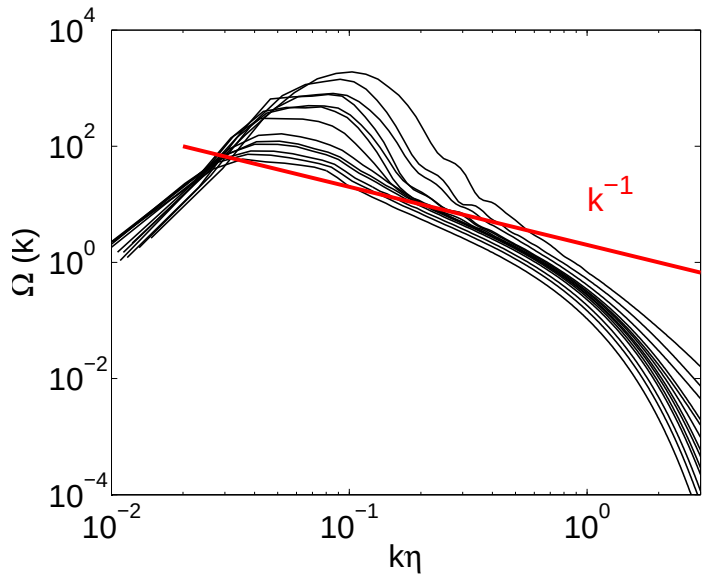
$$S_3 \sim \varepsilon r$$

$$\boxed{S_p \sim r^{\zeta(p)} \sim (\varepsilon r)^{p/3}}$$



Turbulencia Bidimensional

$$\boxed{E_{\omega\omega} = \Omega(k) \sim k^{-1}}$$



INTERMITENCIA

- **Problema Matemático:** ¿Cual es la geometría del soporte de la singularidad cuando $\nu \to 0$?

- **Kolmogorov (1962):** $u_r \sim (\varepsilon_r r)^{1/3}; \quad \varepsilon_r = V^{-1} \int_{r^3} \varepsilon \, \mathrm{d}V$

- **Mandelbrot, Frisch (1970's):** Fractales; Multifractales

- **Problema Experimental (numérico):** ¿Cual es la estructura de los gradientes de velocidad cuando $Re \gg 1$?

LA HERENCIA DE KOLMOGOROV

- **FÍSICA: CASCADA**, $u_r \sim r^{1/3}$, $S_3 = -4\varepsilon r/5$, ε.
- **INGENIERÍA:** $k - \varepsilon$.
- **PROBLEMAS ABIERTOS:** Intermitencia. Escalado.

$\Rightarrow$ **JUSTIFICACIÓN FORMAL** $\Leftarrow$

ESTRUCTURA DETERMINISTA

Corrsin (1960), Roshko (1970), Kline (1975), Lumley (1980) ...

- Experimentos
- Simulación Numérica
- Sistemas Dinámicos
- Control