# Domain Decomposition and Variational Methods

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## Abstract

There are several reasons to study Domain Decomposition Methods (DDM)

- For Parallel Processing, because it is an easy way to allocate tasks to processors in a multi-processor machine.
- For local mesh refinements, by putting a numerical zoom around the regions in which one wants a better precision.
- For new applications such as thermal studies in buildings, or preliminary studies for car for instance in the art design departments where constructive solid geometry is prefered to engineering CAD systems.

For the second item Professor J.L. Lions suggested a method where by the space of the variational formulation is decomposed. For example for the Dirichlet problem in $\Omega$

$$\int_\Omega \nabla u \nabla \hat{u} = \int f\hat{u} \quad \forall \hat{u} \in H_0^1(\Omega)$$

one would seek a solution in the form $u = u_1 + u_2$ such that $u_i \in H_i$ and

$$\int_\Omega \nabla(u_1 + u_2)\nabla(\hat{u} + \hat{u}_2) = \int f(\hat{\hat{u}} + \hat{u}_2) \quad \forall \hat{u}_i \in H_i$$

for which an obvious choice is

$$H_i = H_0^1(\Omega_i) \quad \text{with } \Omega_1 \cup \Omega_2 = \Omega, \quad \Omega_1 \cap \Omega_2 \neq \emptyset$$

It turns out that this method is used in computational fluid dynamics in another form and known as *Chimera* and it is also another version of the Schwarz algorithm for DDM. Convergence was shown by J.L.Lions, error estimation after discretization by Brezzi et al.
We shall also present here a general 3d PDE solvers based on this method, written in C++ for genericity: `freefem3d`; it reads `POV-Ray` geometries, it uses `bison` to parse the the user input in a dedicated programming language with the fictitious domain method in mind for the partial differential equations and the visualization is done with `dx` or `medit` (everything being freeware).

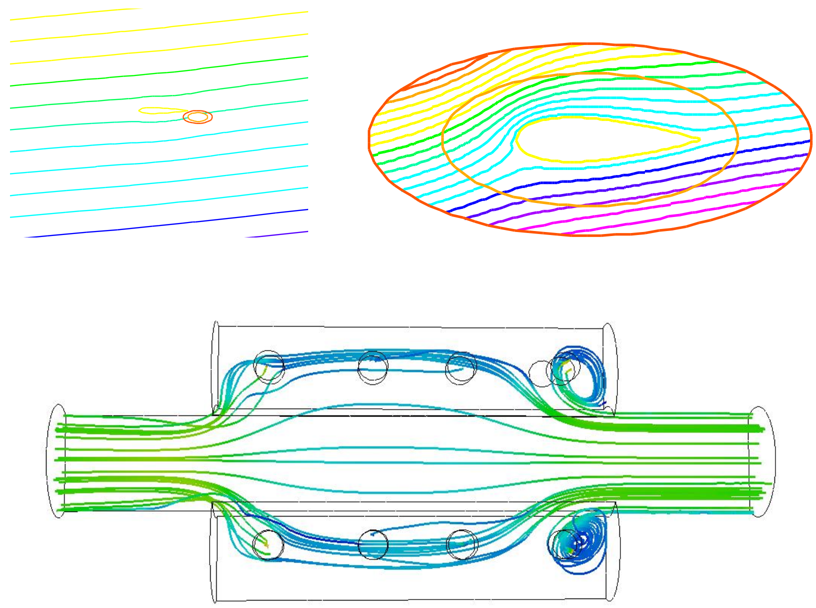

Figure 1: *Stream function around a two-pieces airfoil, namely solution of a Dirichlet problem by the Chimera method converged in 4 iterations. Below, solution by* `freefem3d` *of the Navier-Stokes equations in 3D in a metro station when the train enter.*

# References


[1] Brezzi,F., Lions, J.L., Pironneau, O. (2001): Analysis of a Chimera Method. C.R.A.S., **332**, 655-660

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[3] Steger J.L. (1991): The Chimera method of flow simulation. Workshop on applied CFD, Univ. of Tennessee Space Institute