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A LANGUAGE FOR THE CONSTRUCTION OF PREFERENCES UNDER UNCERTAINTY

(Expected utility/prospect theory/target-based decisions/choice anomalies/benchmarking)

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ABSTRACT

This paper studies a target-based procedure to rank lotteries that is normatively and observationally equivalent to the expected utility model. In view of this equivalence, the traditional utility-based language for decision making may be substituted with an alternative target-based language. Switching language may have significant modelling consequences. To exemplify, we contrast the utility-based viewpoint of prospect theory against the target-based viewpoint and provide an explanation of Allais' paradox based on context dependence instead of distorted probabilities.

1. INTRODUCTION

Suppose that a greedy agent must rank n monetary lotteries $X_1, X_2, ..., X_n$. The agent does not know how to compare two lotteries, so he must use some ranking procedure. Here is a possible one. The agent selects a target t and ranks a lottery X by the probability $P(X \geqslant t)$ that it meets the target; see Manski (1988). However, the agent may not know for sure which target he should select. Then he could replace the sure target t with a random variable T representing his uncertain target and rank a lottery X by the probability $P(X \geqslant T)$ that it meets his uncertain target. We call this the target-based procedure.

Another possible ranking procedure is based on the expected utility model. The agent selects a utility function U over money and ranks a lottery X by its expected utility $EU(X)$. We call this the utility-based procedure. For future use, note that the utility function U is unique up to increasing affine transformations: we say for short that U is cardinal.

A natural question is which one of the two procedures is better. The answer, however, depends on what we mean by «better». One possible approach is to take a normative point of view and interpret «better» as «more rational». Another approach is to consider revealed preferences and interpret «better» as «closer to observed choice behavior». Thus, we may ask two different questions.

Which one of the two procedures is more rational? Which one is closer to the observed choice behavior? Both questions have the same surprising answer: neither one —they are equivalent! If an agent applies the target-based procedure, he behaves *as if* he is maximizing the expected value of a utility function. Vice versa, if he follows the utility-based procedure, he acts *as if* he is maximizing the probability to meet an uncertain target.

We show below why the two procedures are both mathematically and observationally equivalent. For the moment, just note that this implies that any axiomatic foundation for the utility-based procedure works as well for the target-based procedure. Analogously, any choice behavior which can be rationalized by the expected utility model fits equally well the target-based model.

These two equivalence statements would seem to leave no room for interesting questions. There are two different procedures, but only one basic model for decision making.

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Yet, there is one important difference that we should explore. Many, if not all, nonexpected utility models have been suggested as ways to amend the expected utility model against a mounting contrary empirical evidence. Most of these models maintained the notion of a cardinal utility function or, more generally, were framed in a utility-based language. Instead, the target-based model assumes no comprehension of cardinal utilities: it is phrased in a language that requires only an understanding of probabilities.

The target-based approach and the utility-based approach invoke two different languages. Which languages is chosen to amend the basic model (be it expected utility or the equivalent target-based procedure) may affect the descriptive power and the plausibility of our models.

To exemplify, consider the prospect theory put forth in Kahneman and Tversky (1979), which is still one of the best and most complete descriptive models for decision making under risk. Prospect theory was built in three steps, heavily inspired by the expected utility model. First, Kahneman and Tversky amassed a tremendous amount of empirical evidence and compared it with the predictions of the expected utility model. A good chunk of the evidence was compatible, while the rest led to the so called choice anomalies. Second, they fit the compatible evidence coming up with a characteristic shape for the cardinal utility function. Third, they modified some pieces of the expected utility model to fit the choice anomalies, ending up with a nonexpected utility model based on the distorted probabilities suggested first in Edwards (1955, 1962).

To use an analogy, Kahneman and Tversky's prospect theory did for expected utility what Ptolemy's epicycles did for the geocentric theory. But what would happen if prospect theory would be worked out using a target-based language? Maybe we might have an *explanation* for the choice anomalies more convincing than Kahneman and Tversky's story about probability distortions. And if this were the case, the target-based language should be deemed descriptively richer than the utility-based language.

This paper studies whether the target-based language can stake the claim of being descriptivley richer than the utility-based language. We test its power against Kahneman and Tversky's prospect theory. We are well aware that there are many competitors in the race to offer better descriptive models, including the cumulative prospect theory later developed in Kahneman and Tversky (1992). However, prospect theory has a few advantages that make it the ideal benchmark: it is simple to explain, it is more widely known and it misses none of the essential elements that should enter into a descriptive theory of decision making.

The rest of the paper is organized as follows. Section 2 establishes the equivalence of the target-based model and of the expected utility model. Section 3 summarizes prospect theory and discusses the key descriptive assumptions that it imposes on the utility function. Section 4 applies the target-based language to provide an explanation for these assumptions. Section 5 reviews the choice anomalies and how prospect theory deals with them. Section 6 applies the target-based language to this experimental evidence and provides an alternative descriptive theory. Section 7 compares the advantages of the target-based language versus the advantages of the utility-based language and draws some conclusions.

2. TWO EQUIVALENT PROCEDURES

The purpose of this section is to establish the equivalence of the target-based procedure and the utility-based procedure. For simplicity we discuss only the case of decision making under risk, where the outcomes are monetary and the probability distributions are already known to the agent. See Castagnoli and LiCalzi (1996) for arbitrary prizes under risk and Bordley and LiCalzi (1999) for arbitrary prizes under uncertainty in the setting of Savage's (1954) theory of subjective expected utility.

Some formalities will be useful. Let $Y \subset \mathbb{R}$ be a nonempty set of monetary outcomes and let $\mathcal{L}$ be the set of all lotteries on Y. The set Y is completely preordered by the «greater than» preference relation $\geqslant$, which represents a greedy agent. If X is a lottery in $\mathcal{L}$ and F is its cumulative distribution function (c.d.f.), we write $X \sim F$. Given an outcome y in Y, we denote by y^* the degenerate lottery in $\mathcal{L}$ yielding y for sure.

We show that the target-based procedure and the utility-based procedure are equivalent by proving that they are equivalent to a third (apparently) more general ranking procedure; see Churchman and Ackoff (1954). A ranking procedure induces a preference relation $\succcurlyeq$ on $\mathcal{L}$. There are many ways to describe a ranking procedure, but the simplest one is to define a value function: $v: \mathcal{L} \to \mathbb{R}$ and rank $X_1 \succcurlyeq X_2$ if and only if $v(X_1) \geqslant v(X_2)$. Any value function v represents a ranking procedure over $\mathcal{L}$. We assume that the ranking given by v is consistent with the greedy preference relation $\geqslant$ on Y: $y_1 \geqslant y_2$ if and only if $v(y_1^*) \geqslant v(y_2^*)$.

We consider the class of ranking procedures associated with value functions that are (weakly continuous and) linear in the probability distributions; that is, given $X \sim F$, the value function v can be written as

$$v(X) = \int_x W(x)\, dF(x) \qquad (1)$$

where $W(x) : Y \to \mathbb{R}$ is a real-valued, bounded, continuous, increasing, and cardinal weight function. As it is well-known, the independence axiom and the continuity of $\succcurlyeq$ (in the weak topology) are a necessary and sufficient condition for the existence of a linear ranking procedure; see for instance Theorem 3 in Grandmont (1972). This characterization result, however, is conspicuously silent about how we should interpret the weight function W.

To provide an interpretation, we need to turn to a language. Let us bring in the two procedures described in the introduction. Following the target-based model, the agent must first subjectively assess the c.d.f. $P(x \geqslant T)$ of his uncertain target T, which we assume stochastically independent of the lotteries in $\mathcal{L}$. Then, he evaluates a lottery $X \sim F$ using the ranking procedure associated with the value function

$$v_1(X) = P(X \geqslant T) = \int_x P(x \geqslant T)\, dF(x).$$

This ranking procedure coincides with the class in (1) because, since $W(x)$ is bounded and cardinal, we can normalize its range to [0, 1] and let $P(x \geqslant T) = W(x)$. The target-based procedure is a linear ranking procedure, where the weight function $W(x)$ is interpreted as the c.d.f. $P(x \geqslant T)$ of an uncertain target T.

Following the expected utility model, the agent must first subjectively assess his cardinal utility function $U : Y \to \mathbb{R}$. Then, he evaluates a lottery $X \sim F$ using the ranking procedure associated with the value function

$$v_2(X) = EU(X) = \int_x U(x)\, dF(x) \qquad (2)$$

Again, if we let $U(x) = W(x)$, this ranking procedure coincides with the class in (1). The expected utility procedure is also a linear ranking procedure, where the weight function $W(x)$ is interpreted as the *cardinal* utility function $U(x)$.

Each of the two interpretations needs some exogenous component, which is left for the agent to be subjectively assessed. The target-based language requieres a stochastically independent uncertain target T. The utility-based language requires a utility function that is unique up to affine increasing transformations. In our opinion, neither requirement can claim to be more plausible than the other. And, in any case, both conform to (1); therefore, the two procedures share the same axiomatic foundations and are observationally equivalent.

How do we move from one procedure to the other? We can bypass the weight function $W(x)$ and check directly when v_1 and v_2 define the same ranking. After a normalization, the two equalities $P(x \geqslant T) = W(x) = U(x)$ must hold. Hence, the two procedures are equivalent if we let

$$P(x \geqslant T) = U(x).$$

To put it differently, the equivalence follows if we think of the «old» cardinal utility of x as the probability that the uncertain target T is not greater than x: that is, if we interpret $U(100)$ as the probability that the agent's target is not greater than 100 euros, rather than as the cardinal utility of 100 euros for the agent.

This somewhat suprising equation is the major piece of the «dictionary» to translate the target-based language into the utility-based language and vice versa. We use this translating device throughout the rest of the paper; see Berhold (1973), Borch (1968) or Castagnoli and LiCalzi (1996) for a few (mutually independent) excursions on this theme.

3. PROSPECT THEORY AND THE UTILITY FUNCTION

The major purpose of this paper is to compare the potential descriptive power of a target-based language versus the sucesses of the utility-based language. To provide material for this comparison, this section and Section 5 review the major propositions of the prospect theory developed by Kahneman and Tversky (1979) to account for the empirical evidence against the expected utility model. For a broader perspective on prospect theory and behavioral decision theory see Thaler (1987).

Prospect theory deals with decision making under risk, where the problability distributions for the lotteries are known to the agent. The theory is developed only for monetary lotteries with finite support. To ensure maximum consistency, we restrict attention to finite lotteries over money and, in this section, we strictly adhere to a utility-based language.

Prospect theory has four major assumptions. The first one is that there is a preliminary editing phase, during which outcomes and probabilities of the lotteries may be transformed. Typical phenomena that may occur during this phase are the coding of outcomes as gains and losses, the segregation of riskless components or the rounding of probabilities. The editing phase is crucial to understanding how the agent perceives a lottery. Much of what goes on in a ranking task probably takes place at this stage. However, since the editing phase is carried out before any cardinal utility function enters the picture, we do not need to examine it in greater detail.

The other three major assumptions are (i) there exists a utility function U over outcomes; (ii) there exists a probability distorsion function π which describes how the agent perceives (or weighs) the known probabilities: more precisely, if a lottery X has the probability distribution p, the probability p_i of an outcome x_i occurring is perceived as $\pi(p_i)$; (iii) there exists a ranking procedure which combines utilities and (distorted) probabilities.

Prospect theory assumes that the ranking procedure is linear in the distorted probabilites. In other words, the ranking procedure is generated by the value function

$$v(X) = \sum_x U(x)\pi[p(x)] \qquad (3)$$

which is linear in π but not in p. Therefore, prospect theory postulates a model which in general is not linear in the known probabilities. Given the similarity of (3) to a linear ranking procedure, it should be apparent how little prospect theory tries to part away from the expected utility model.

Based on the assumption that the ranking procedure is linear in the distorted probabilities, Kahneman and Tversky (1979) examines the empirical evidence and deduces what properties U and π should satisfy to make (3) compatible with it. In the rest of this section, we consider what prospect theory has to say about U.

The utility function

Prospect theory summarizes the empirical evidence about the utility function U in three effects that have a clear psychological interpretation:

(i) Lack of asset integration: people are concerned about changes with respect to some reference point, rather than about their final state of wealth.

(ii) Reflection effect: the marginal impact of both positive changes (gains) and negative changes (losses) decreases with their magnitudes.

(iii) Loss aversion: losses loom larger than gains of equivalent amount.

Both Kahneman and Tversky (1979) and the subsequent literature have qualified these propreties in many ways. In particular, the empirical evidence which supports them is not always as clear-cut as one might wish. Overall, however, these properties are a robust summary of many independent experiments.

The following proposition states how they can be formalized and made to fit both the expected utility model in (2) and the prospect theory of (3). Given a reference point r, we call gains and losses those outcomes that are respectively coded as positive or negative changes (with respect to r).

Proposition 1. *Given either the expected utility model or the prospect theory, the following three characterizations hold.*

(i) *Lack of asset integration holds if and only if the utility function U is defined over changes with respect to some reference point.*

(ii) *The reflection effect holds if and only if U is concave over gains and convex over losses.*

(iii) *Loss aversion holds if and only if U is steeper over losses than over gains.*

This characterization is a descriptive result, stating which properties of U must be assumed to make (2) or (3) compatible with the experimental findings. However, the proposition does not tell us why the three effects occur. What brings about lack of asset integration, the reflection effect and loss aversion is accounted for but not explained.

4. A TARGET-BASED EXPLANATION

We can apply the target-based language to offer an explanation for all the three effects described in the previous section. Our purpose is not to derive an alternative mathematical theorem: Proposition 1 accounts for the experimental evidence in an elegant and simple way. Our intent is to *explain* what may bring about precisely the propreties described by Proposition 1. We already know the «what?»; this section looks at the «why?».

We begin with the explanation for the lack of asset integration suggested by the target-based language. Suppose for a moment that the agent has a known target. When he evaluates an outcome, there is a natural sense in which this is good or bad: it meets the target or it does not. The good outcomes are coded as gains and the bad outcomes are coded as losses. When the target is uncertain, the mental process of pitting an outcome against an uncertain target is still dichotomous: if an outcome represents a change that improves his chances of meeting the target, the agent codes it as a gain; otherwise, as a loss. Proposition 1.(i) is the mathematical representation of a classification task.

We now move to the reflection effect. Assume for simplicity that the uncertain target T has a probability density $\tau(x)$ and consider which kind of probability density for the target would generate the reflection effect. The reflection effect states that $P(x \geqslant T) = U(x)$ is concave over gains and convex over losses. Since $\tau(x)$ is the derivative of the c.d.f. $P(x \geqslant T) = U(x)$, the probability density would have to be decreasing over the domain of gains and increasing over the domain of losses; that is, τ should be unimodal around the reference point. See Figure 1, where the modal target is coded as the reference point (conventionally, the 0 outcome).

Hence, the reflection effect follows from a subjective assessment that there is one modal outcome for the uncertain target (which acts as a reference point) and that

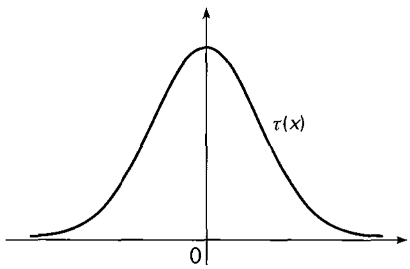

Figure 1. A unimodal probability density for the target.

the probability of the target being different from the reference point is decreasing as we move away from it. Proposition 1.(ii) is the mathematical representation of a probability judgement.

As Kahneman and Tversky (1979) diffusely point out, the reference point used to code outcomes may differ from the *status quo* or may shift over time. In the target-based language, we equate the reference point with the modal outcome of the distribution. Therefore, the mode of the uncertain target may differ from the *status quo* or may shift over time. This is plausible and consistent with the assumption that the target is subjectively assessed. Depending on the structure of the problem, we may expect that the most likely target is different from the current outcome. And as we obtain more information, we may update the distribution of the target so that the modal outcome would shift over time.

Incidentally, it is worthwhile to pause and note which kind of probability distribution for the target would generate risk averse behavior over all lotteries. Since risk aversion follows from a concave $U(x) = P(x \geqslant T)$, this implies that the density function τ should be decreasing; see Figure 2. Hence, risk aversion follows from a conservative evaluation which «ascribes high probability to the uncertain» target being a low outcome. This offers an explanation for which psychological factors may lay behind the characterization of risk aversion as concavity of the cardinal utility function.

The third and last effect to examine is loss aversion under risk, which states that $P(x \geqslant T) = U(x)$ has a higher

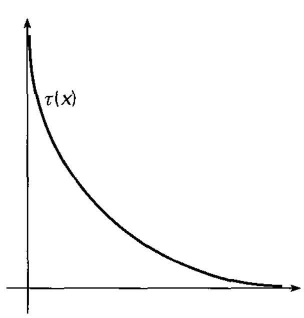

Figura 2. A conservative assessment of the target.

derivative over losses than over gains of equivalent amount. This property has an ambiguous interpretation, because Kahneman and Tversky (1979) does not specify the admissible range of gains and losses. If this range is $\mathbb{R}$, loss aversion implies $U'(x) < U'(-x)$ for all $x > 0$. On the other hand, if losses are bounded below by $-b$, it suffices that $U'(x) < U'(-x)$ for all x in $(0, b)$. The picture on p. 279 of Kahneman and Tversky (1979) and the literature suggest the second interpretation. Although the target-based language may accommodate either case, we also adopt this second interpretation because it is more ralistic: losses are usually bounded below, at worst by bankruptcy.

Given this interpretation, since $\tau(x)$ is the derivative of the c.d.f. $P(x \geqslant T) = U(x)$, the reflection effect requires $\tau(x) < \tau(-x)$ over some (possibly large) interval $(0, b)$ of the reference point. This implies that the probability density for the target should be asymmetric around the modal outcome; see Figure 3 for two examples.

Loss aversion follows from a subjective judgement that expects targets just below the reference point to be more likely than those just above it. Proposition 1.(iii) is the mathematical representation of a prudential attitude in the evaluation of the uncertain target.

There is a variety of distributions that may be consistent with this prudential attitude. The picture on the left of Figure 3 shows that the probability of the target being very high must not necessarily be small: for example, a college student may sets her reference point equal to her low current endowment, while still nourishing great expectations about herself. On the other hand, the picture on the right shows that the probability of the target being high can be small: for example, an established banker may feel that there is no much room left to improve on his reference point. These two cases would exhibit markedly different values for the slopes of $U(x)$ in a neighborhood of 0.

In spite of its intuitive plausibility, risk seeking behavior over losses has received less empirical support than the other two effects; for instance, Bernstein et alii (1997) do not find any evidence of it. The variety of compatible distributions for the target suggests that some experiments may have failed to recognize it only because their design could not take into account differences across people in the assessments of their targets. It is to be hoped that this target-based conjecture will be put under experimental testing.

5. CHOICE ANOMALIES AND DECISION WEIGHTS

In this section we go back to prospect theory and to a utility-based language. Whichever their explanation may

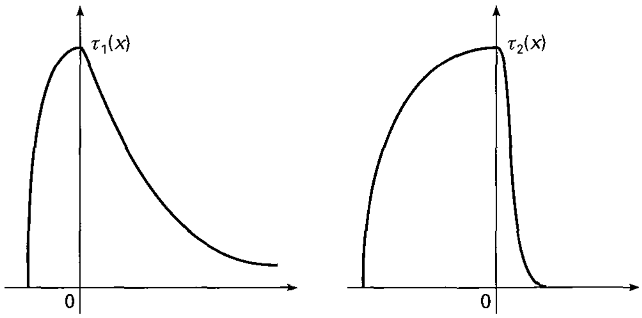

Figure 3. Two asymmetric assessments of the target.

be, the mathematical properties of the utility function collected in Proposition 1 do not suffice to account for all the empirical evidence. Kahneman and Tversky (1979) report a few choice anomalies that invalidate the expected utility model in (2), but may be accommodated by introducing the distorted probabilities of the nonlinear ranking procedure in (3). To convey the flavor of their argument, it will be enough to look at four experiments chosen from the rich corpus of choice anomalies.

Allais' paradox

The first experiment is the well-known Allais' paradox. We recall it in one of its versions, dubbed as Problems 1 and 2 in Kahneman and Tversky (1979). When offered to choose between the two lotteries

$$A = \begin{cases} 2.500 & 0,33 \\ 2.400 & 0,66 \\ 0 & 0,01 \end{cases} \quad \text{and} \quad B = \{2.400 \quad 1, \tag{4}$$

most people (82 %) pick B. However, when offered to choose between the two lotteries

$$C = \begin{cases} 2.500 & 0,33 \\ 0 & 0,67 \end{cases} \quad \text{and} \quad D = \begin{cases} 2.400 & 0,34 \\ 0 & 0,66 \end{cases} \tag{5}$$

most people (83 %) pick C. These two modal choices together are incompatible with the expected utility model.

Allais' paradox has stimulated many nonexpected utility models that can account for this choice anomaly; see for instance Machina (1982) and Gul (1991). Most of these models share the intuition that people seem to give a disproportionate weight to lottery B, because it offers a sure gain of 2.400. For this reason, this choice anomaly is also known as the certainty effect.

The certainty effect occurs even with two-outcome gambles. For instance, consider Problems 3 and 4 in Kahneman and Tversky (1979). When offered to choose between the two lotteries

$$A = \begin{cases} 4.000 & 0,80 \\ 0 & 0,20 \end{cases} \quad \text{and} \quad B = \{3.000 \quad 1, \tag{6}$$

most people (80 %) pick B. However, when offered to choose between the two lotteries

$$C = \begin{cases} 4.000 & 0,20 \\ 0 & 0,80 \end{cases} \quad \text{and} \ D = \begin{cases} 3.000 & 0,25 \\ 0 & 0,75 \end{cases} \tag{7}$$

most people (65 %) pick C. Again, the two modal choices together run against the expected utility model.

In fact, the certainty effect is so robust that it persists even in a setting where lottery B offers only a very likely gain, but not the certainty of it. For example, in Problems 7 and 8 in Kahneman and Tversky (1979), the usual pattern of B (86 % of choices) and C (73 % of choices) is replicated when people are respectively offered a choice between

$$A = \begin{cases} 6.000 & 0,45 \\ 0 & 0,55 \end{cases} \quad \text{and} \quad B = \begin{cases} 3.000 & 0,90 \\ 0 & 0,10 \end{cases} \tag{8}$$

and between

$$\text{C} = \begin{cases} 6.000 & 0,01 \\ 0 & 0,999 \end{cases} \quad \text{and} \quad D = \begin{cases} 3.000 & 0,02 \\ 0 & 0,998 \end{cases} \tag{9}$$

This led to an empirical generalization of the Allais' paradox known as the common ratio effect: suppose that a lottery offering y with probability q (and nothing otherwise) is deemed indifferent to another lottery offering $x < y$ with probability $p > q$ (and nothing otherwise); then, for $0 < r < 1$, a third lottery offering y with probability qr is preferred to a fourth lottery offering x with probability pr.

The reverse Allais' paradox

A puzzling companion to the traditional Allais' paradox is the reverse Allais' pradox, which occurs when we change the sign of all the outcomes in the original formulation. For instance, take the two pairs of lotteries in (6) and (7) and change the sign of all the outcomes. These are Problems 3' and 4' in Kahneman and Tversky (1979).

Now, when offered to choose between the two lotteries

$$A' = \begin{cases} -4.000 & 0,80 \\ 0 & 0,20 \end{cases} \quad \text{and} \quad B' = \{-3.000 \ 1 \tag{10}$$

most people (92 %) pick A'. Instead, when offered to choose between the two lotteries

$$C' = \begin{cases} -4.000 & 0,20 \\ 0 & 0,80 \end{cases} \quad \text{and} \quad D' = \begin{cases} -3.000 & 0,25 \\ 0 & 0,75 \end{cases} \tag{11}$$

most people (58 %) pick D'. Preferences are reversed when we transform gains into losses. This is remarkable because we rarely observe a paired choice of A and D when the Allais' paradox is formulated over gains. In the words of Kahneman and Tversky (1979), «certainty increases the aversiveness of losses as well as the desiderability of gains».

Distorted probabilities

None of the four choice anomalies mentioned is compatible with a linear ranking procedure like the expected utility model. If we want a model that can account for them, we need to alter some feature of the linear ranking procedure. Kahneman and Tversky (1979) followed Edwards (1955, 1962) and took the route of altering the probabilities with a probability distorsion function π. This function maps the known probability p into a different value $\pi(p)$, still in [0, 1]. The distorted probabilities, called «decision weights», may not obey the probability axioms and should not be interpreted as alternate subjective probabilities.

The introduction of decision weights in conjunction with the ranking procedure in (3) can account for many choice anomalies. For example, the Allais' paradox of (4)-(5) can be generated by the property that very low probabilities are overweighted; that is, $\pi(p) > p$ if p is small. Or, more generally, the common ratio effect can be obtained if the decision weights satisfy the subproportionality property that $\pi(q)/\pi(p) \leqslant \pi(qr)/\pi(pr)$, for all $p > q$ and r in (0, 1).

It is not necessary to delve into the technicalities of decision weights to make our point. If a reasonably complete «explanation» of all the choice anomalies listed in Kahneman and Tversky is to be found in the introduction of decision weights, the distorsion function has to be very complicated; see Prelec (1998). For instance, Kahneman and Tversky (1979) end up assuming that π satisfies five properties so stringent that, if we add the requirement that the distorsion function π be continuous, it is impossible to satisfy them simultaneously. This raises two problems.

The first one is that, whereas the clarity and elegance of the assumptions on the utility function are obvious, the distorsion function seems convoluted. The obvious reply is that a descriptive model does not have to be simple: it has to work. To follow up the analogy in the introduction, this is the argument used to defend Ptolemy's epicycles.

The second major problem is that what decision weights represent or how they should be interpreted is left unexplained. Silence on this problem reigns even in Hogarth and Einhorn (1990), whose stated purpose is to complete prospect theory by giving a descriptive model of how people assess decision weights for probabilities. Nor has any light on this problem been shed by the huge literture dealing with nonlinear ranking procedures based on distorted probabilities, including the well-known class of rank-dependent utility models initiated by Quiggin (1982) and Yaari (1987).

6. A TARGET-BASED DESCRIPTIVE THEORY

This section completes Section 4 by providing a target-based descriptive theory that explains the experimental evidence presented in Kahneman and Tversky (1979). Our purpose is to show that two simple modelling tools can account for all choice anomalies of Section 5. Therefore, combining the editing phase from prospect theory, the target-based explanation of Proposition 1 from Section 4 and these two tools, we obtain an alternative theory with the same descriptive power of prospect theory.

Context-dependence

Let us go back to the Allais' paradox describe in (4) and (5) in Section 4. Its standard explanation has two parts. First, since most people are risk-averse, the modal choices should be A over B and C over D. Thus, in some sense, the paradox lies in the choice of B. The second part aims to explain why most subjects choose B.

When comparing lotteries A and B, people tend to give a disproportionate weight to lottery B. In Kahneman and Tversky's (1979) words, «people overweight outcomes that are considered certain, relative to outcomes which are merely probable». Prospect theory does not capture this intuition because it accounts for Allais' paradox by assuming that the small probability of obtaining 0 in lottery A is overweighted, rather than by overweighting the sure payoff of 2.400 in B. Based on the target-based language, we can offer a model for the second part of this explanation which is closer to intuition.

The agent who comes to the laboratory has some kind of uncertain target in mind. For instance, he has expectations about how much he might win (or be paid). Unless there is a contrary reason, he assesses the loteries he is offered using this uncertain target. However, if the context provides a strong cue, he may update the prior distribution of the target and use the posterior distribution for the ranking.

We believe that context-dependence is the leading force behind Allais' paradox. When one of the feasible lotteries offers a sure gain of 2.400, the agent takes this into account and revises his prior distribution for the target. If there is money to be made for sure, this salient piece of information may be used to update the initial assessment of the target. Consistent with intuition, the updating should increase the probability that the uncertain target is 2.400.

How do we model this context-dependence of the uncertain target? From a normative point of view, this would require setting up a Bayesian problem in which the uncertain target T has a prior distribution $P_0(x \geqslant T) = F_T^0(x)$ that is updated into a posterior distribution $P_1(x \geqslant T) = F_T^1(x)$ by using the information contained in the pair of lotteries (4) which is presented to the agent. This could be made in many ways, but probably they would all be too complicated to serve the simple descriptive purpose we are after. Therefore, we suggest a much simpler model that captures the essential features of Bayesian updating.

Let F_T^0 be the prior c.d.f. of the target and let $F_1(x)$ and $F_2(x)$ be respectively the c.d.f. of the first and the second lottery in the pair. For instance, if the pair of lotteries offered is (4), $F_1(x)$ is the c.d.f. of lottery A and $F_2(x)$ is the c.d.f. of lottery B. Let $F_T^1(x)$ be the posterior distribution of the target, given the pair of lotteries offered to the agent. Then F_T^1 should depend on the prior F_T^0 and on the contextual distributions F_1 and F_2.

We assume that the posterior distribution F_T^1 is a convex combination of F_T^0, F_1 and F_2:

$$F_T^1(x) = \alpha_0 F_T^0(x) + \alpha_1 F_1(x) + \alpha_2 F_2(x) \tag{12}$$

with $\alpha_0 + \alpha_1 + \alpha_2 = 1$ and $\alpha_i \geqslant 0$ for $i = 1, 2, 3$. We also assume that the lotteries offered are evaluated using a stochastically independent target T distributed according to F_T^1. Note that the lotteries offered may affect the distribution of the target; however, once the posterior distribtion is obtained, the ranking procedure is still linear because of the stochastic independence of T.

Even if simple, the updating rule in (12) offers many degrees of freedom. For convenience, we make two assumptions which entail no loss of generality. We explain these assumptions with reference to the Allais' paradox of Problems (4) and (5), but they are also used throughout the rest of the paper. First, we assume that the support of the prior distribution of the target is the interval between the minimum and the maximum outcome across all offered lotteries; that is [0, 2.500]. Second, we assume that without context-dependence the agent would be risk-neutral; that is, the prior distribution $F_T^0(x) = U(x)$ is linear over its support. This linearity amounts to saying that that the prior c.d.f. $F_T^0$is uniform.

Appendix A.2 shows that relaxing these assumptions by assuming a larger support or some degree of risk aversion would only make the choice of A over B less likely. Therefore, the «paradoxical» choice of B cannot be an artifact of these two assumptions. It is also easy to check that our assumptions imply that C is preferred to D.

The context-dependent explanation of the Allais' paradox is that the posterior distribution F_T^1 puts a substantial weight on the lottery B which offers a sure gain of 2.400. In our model, the weight on lottery B is α_2. Assuming that the agent follows a target-based procedure where the posterior distribution of the uncertain target is given by (12), Appendix A.1 shows that the agent prefers B to A if the α's satisfy the restriction $(0{,}0036)\alpha_0 \leqslant (0{,}01)\alpha_2 - (0{,}1023)\alpha_1$.

Assuming that lottery A has no contextual weight, let $\alpha_1 = 0$. Then $\alpha_0 = 1 - \alpha_2$. Substituting, we obtain that B is preferred to A for $\alpha_2 \geqslant (0{,}36)/(1{,}36) \approx 0{,}2647$. That is, if the contextual weight of B is at least 0,27, then people would choose B over A. An increase of about 27 % in the probability that the uncertain target is at least 2.400 can explain the anomaly in Allais' paradox. The necessary increase would be even lower if we assumed risk aversion or a larger support for the prior target.

Contrast this with the explanation based on the distorted probabilities of prospect theory. It accounts for the choice of B by assuming that the probability 0,01 of winning 0 in lottery A is distorted to $\pi(0{,}01) > 0{,}01$, which makes A less appealing. While the target-based explanation stresses the salience of B, prospect theory opts to downgrade the competing alternative.

This target-based explanation is robust. For instance, if we repeat a similar argument for (6) we find that $\alpha_2 \geqslant 0{,}2$ suffices to explain the choice of B; see Appendix A.3. In fact, all the anomalies reported in Kahneman and Tversky can be explained by a value of α_2 not higher than the $\alpha_2 = 27\,\%$ found above for the Allais' paradox.

The explanation also applies to cases like (8) and (9), where any risk-neutral agent is indifferent over the lotteries in each pair. Assuming any degree of (strict) risk aversion, we should expect the choice of B over A and D over C. Therefore, the source fo the anomaly here is the choice of C. By (16) in Appendix A.4, this choice would follow if the contextual weights are such that $\alpha_1 \geqslant 2\alpha_2 + K$, where K is a suitable positive constant. The contextual importance of C should be more than twice as large as D's. This result is consistent with the intuition that people probably find C salient because it associates a richer outcome with a very low probability of winning.

Avoiding losses

Besides context-dependence, a descriptive explanation of the reverse Allais' paradox of (10) and (11) should incorporate an assumption analogous to loss aversion. The target-based procedure should be slightly different when the problem is framed exclusively in terms of gains or of losses. When dealing with gains, we assume that barely making the target is good and therefore that the agent tries to maximize $P(X \geqslant T)$. Instead, when dealing with losses, we assume that just making the target is bad and thus that the agent tries to maximize $P(X > T)$ or, equivalently, to minimize $P(X \leqslant T)$ or $P(X < T)$ would be equivalent.

Under this slightly modified target-based rule, we can apply the context-dependent model used before to account for the choice of A' in (10) and of D' in (11); see Appendix A.5. Under our usual simplifying assumptions, the choice of A' over B' can be explained exactly by the same equation already obtained for (6): if the contextual weight of B' is at least 0,2, the salience of the certain outcome in B' leads the agent to prefer A' in the attempt to prevent a sure loss.

7. COMPARING LANGUAGES

A large part of decision theory is framed in a utility-based language; see Rubinstein (1988) for a notable exception. There is no doubt that this language has led to many successes and there is no question about its importance, especially from a normative viewpoint. This section tries to assess the potentialities of the target-based language for the theory of decision making against the benchmark of the utility-based language.

How do we judge if a language A is better than another language B? The answer depends on our purposes, but the following four criteria should be part of the answer:

(i) Expressiveness: is A at least as powerful as B for our purposes? In particular, can it fit everything we can say in the old language?

(ii) Ease of use: is A at least as easy to learn and use azs B?

(iii) Explanatory power: does A lead to new concepts or to «better» explanations?

(iv) Relevance: can A handle interesting problems? In particular, can it handle problems that are relevant to economics?

We evaluate the target-based language on the basis of the first three criteria and advance some suggestions about the fourth criterion.

Expressiveness. As shown in Section 2, the target-based language and the utility language have the same mathematical description. Therefore, anything that can be formalized in one language can also be formalized in the other one. For example, the normative foundations of expected utility equally apply for the target-based procedure. As a formal language, therefore, the target-based language is at least as expressive as the utility-based language.

Ease of use. The target-based language is phrased entirely in the language of probability. Since it does not require an understanding of cardinal utilities, it is simpler to explain and to use. For example, instead of estimating $U(x)$ using the standard utility-based elicitation procedures, we might ask the agent to draw the density function for his target and estimate his c.d.f. from there. For another example, consider the problem of interpersonal comparison of cardinal utilities: what are the implications of $U_1(x) > U_2(x)$? In the target-based language, this difficult question translates into a comparison between the probability that agent 1 attaches to his target being less than x versus the probability that agent 2 assesses for her target being less than x. Since subjective probabilities can be compared, a target-based language may make this problem easier to attack.

Explanatory power. Section 4 and 6 were devoted to show that the target-based language may offer a descriptive theory alternative to prospect theory. For example, we offered a context-dependent explanation for the Allais' paradox as opposed to the distorsion of probabilities characteristic of prospect theory. On this basis, we claim that the target-based language may have at least as much explanatory power as one descriptive theory based on the notion of utility.

Relevance. Judging the relevance (in particular to economics) of a language is a very subjective task. Therefore we will not attempt it here. However, we will try to suggest some problems which a target-based language may model or attack more successfully than a utility-based language. For instance, LiCalzi (1999) estimates bounds for the expected utility of partially known lotteries which lead to simple dominance heuristics over limited domains.

We begin with some modelling issues. We have already argued that if we view $U(x) = P(x \geqslant T)$ as a probability distribution, we can update it on the basis of new information. Therefore, we can model context-dependence as we did in Section 6. Or, we can model learning as the repeated updating of U; see Della Vigna and LiCalzi (1999). More generally, we should be able to model a situation where preferences are path-dependent, in the sense that which targets a person sets for herself depends on her past experiences. Note also that the target-based language is extremely well-suited to deal with the satisficing approach proposed in Simon (1955) for modelling bounded rationality.

Economic problems for which the target-based language may offer interesting suggestions include the following: bargaining (what is my opponent's target?), ultimatum games (acceptance of low payoffs depends on the target), search behavior (acceptance is conditional on the target), purchase of lottery tickets (certainly justifiable when the target is becoming millionaires). A recent paper by Shafir, Diamond and Tversky (1997) looks at money illusion: we conjecture that this occurs when an agent formulates his target in real values but faces lotteries denominated in nominal values.

Finally, the target-based language may lead to a different viewpoint on decision making. The three official approaches in decision theory are nomative (telling people what they should do), prescriptive (helping people to fulfill the normative criteria), and descriptive (accounting for what people actually do); see Bell, Raiff and Tversky (1988). We would like to suggest that these three approaches could be usefully complemented by a fourth *constructive* approach, which should explain how people construct their preferences whenever they do no happen to know them already. Both the target-based procedure and the expected utility procedure in the introduction were discussed in this perspective; see Chapter 5 in Payne et alii (1993) for related ideas.

APPENDIX

A.1. Allais' paradox. Kahneman and Tversky (1979) considers only choices over pairs of lotteries concerning (at most) three elements. Therefore, we can restric attention to choice over two lotteries

$$X = \begin{cases} x_1 & p_1 \\ x_2 & p_2 \\ x_3 & p_3 \end{cases} \quad \text{and} \quad Y = \begin{cases} x_1 & q_1 \\ x_2 & q_2, \\ x_3 & q_3 \end{cases}$$

with $x_1 \geqslant x_2 \geqslant x_3$.

According to the target-based model, when called to choose between X and Y, the agent would maximize the probability of meeting an uncertain target T distributed according to the c.d.f. $U(x)$. For lottery X, this probability is

$$P(X \geqslant T) = p_1 P(x_1 \geqslant T) + p_2 P(x_2 \geqslant T) + p_3 P(x_3 \geqslant T) =$$

$$= \sum_{i=1}^{3} p_i U(x_i)$$

where the last equality follows from $U(x) = P(x \geqslant T)$.

When there is context-dependence, we follow Section 6 and replace $U(x)$ by the posterior c.d.f.

$$F_T^1(x) = \alpha_0 F_T^0(x) + \alpha_1 F_1(x) + \alpha_2 F_2(x) \qquad (13)$$

where F_1 and F_2 are respectively the c.d.f.'s of lottery X and Y. Therefore, the probability that X meets the uncertain target or, for short, its value is

$$\sum_{i=1}^{3} p_i F_T^1(x_i) = \sum_{i=1}^{3} p_i [\alpha_0 F_T^0(x_i) + \alpha_1 F_1(x_i) + \alpha_2 F_2(x_i)].$$

Consider the pair of lotteries A and B in (6). Applying (13), the value of lottery A is

$$(0{,}33)\,[\alpha_0 F_T^0(2.500) + \alpha_1 + \alpha_2] +$$
$$+ (0{,}66)\,[\alpha_0 F_T^0(2.400) + (0{,}67)\alpha_1 + \alpha_2]$$
$$+ (0{,}01)\,[\alpha_0 F_T^0(0) + (0{,}01)\alpha_1]$$

Analogously, the value of B is

$$\alpha_0 F_T^0(2.400) + (0{,}67)\alpha_1 + \alpha_2 .$$

Comparing these two values, B is preferred to A if

$$[(0{,}34) F_T^0(2.400) - (0{,}33) F_T^0(2.500) - (0{,}01) F_T^0(0)].$$
$$\alpha_0 \geqslant (0{,}1023)\alpha_1 - (0{,}01)\alpha_2.$$

Suppose that F_T^0 is uniformly distributed on [0,2.500]. Then $F_T^0(2.500) = 1$, $F_T^0(2.400) = 0{,}96$, and $F_T^0(0) = 0$. Therefore, the inequality becomes

$$-(0{,}0036)\alpha_0 \geqslant (0{,}1023)\alpha_1 - (0{,}01)\alpha_2 \qquad (14)$$

A.2. No loss of generality. Both the assumption that F_T^0 has a support (strictly) including the interval [0,2.500] and that F_T^0 is concave (which corresponds to risk aversion) imply that the left-hand side of (14) would be greater and therefore that a lower value of α_2 would suffice to make B preferred to A. Therefore, we can conclude that a contextual weight for lottery B higher than 0,2647 can explain the choice of B over A in the Allais' paradox under the assumption of Section 6.

A.3. Another pair. Consider the pair of lotteries A and B in (4). Applying (13), the values of A is

$$(0{,}80)\,[\alpha_0 F_T^0(4.000) + \alpha_1 + \alpha_2] + (0{,}20)\,[\alpha_0 F_T^0(0) +$$
$$+ (0{,}20)\alpha_1]$$

and the value of B is $\alpha_0 F_T^0(3.000) + (0{,}20)\alpha_1 + \alpha_2$. Then B is preferred to A if and only if

$$[F_T^0(3.000) - (0{,}80) F_T^0(4.000) -$$
$$- (0{,}20) F_T^0(0)]\ \alpha_0 \geqslant (0{,}64)\alpha_1 - (0{,}20)\alpha_2.$$

For F_T^0 uniformly distributed on [0,4.000], we have $F_T^0(4.000) = 1$, $F_T^0(3.000) = 0{,}75$, and $F_T^0(0) = 0$. Therefore, this inequality becomes

$$-(0{,}05)\alpha_0 \geqslant (0{,}64)\alpha_1 - (0{,}20)\alpha_2 \qquad (15)$$

Letting $\alpha_1 = 0$ and $\alpha_0 = 1 - \alpha_2$, we obtain that B is preferred to A for $\alpha_2 \geqslant (1/5)$.

A.4. A pair of fair gambles. Consider the pair of lotteries C and D in (9). Applying (3), the value of C is

$$(0{,}001)\,[\alpha_0 F_T^0(6.000) + \alpha_1 + \alpha_2] + (0{,}999)\,[\alpha_0 F_T^0(0) + \\ + (0{,}999)\alpha_1 + (0{,}998)\alpha_2]$$

and the value of D is

$$(0{,}002)\,[\alpha_0 F_T^0(3.000) + (0{,}999)\alpha_1 + \alpha_2] + (0{,}998)\,[\alpha_0 F_T^0(0) + \\ + (0{,}999)\alpha_1 + (0{,}998)\alpha_2]$$

Then C is preferred to D if and only if

$$10^5[(0{,}001)F_T^0(6.000) + (0{,}001)F_T^0(0) - (0{,}002) \\ F_T^0(3.000)]\alpha_0 + \alpha_1 \geqslant 2\alpha_2$$

Assuming risk aversion, the term in brackets is negative and therefore we can rewrite this inequality as

$$\alpha_1 \geqslant 2\alpha_2 + K \qquad (16)$$

with $K \geqslant 0$.

A.5. Gambles over losses. Consider the pair of lotteries A' and B' in (10). Instead of using $P(X \geqslant T)$ as the value function for a lottery X, we use $P(X > T)$. This only requires that we substitute the left limit $F(x^-)$ for $F(x)$ whenever a c.d.f. is used in the above formulas. Hence, we replace (13) with

$$F_T^1(x) = \alpha_0 F_T^0(x^-) + \alpha_1 F_1(x^-) + \alpha_2 F_2(x^-).$$

The value of A' is now

$$(0{,}80)\,[\alpha_0 F_T^0(-4.000^-)] + (0{,}20)\,[\alpha_0 F_T^0(0^-) + (0{,}80)\alpha_1 + \alpha_2]$$

and the value of B' is $\alpha_0 F_T^0(-3.000^-) + (0{,}80)\alpha_1$. Then A' is preferred to B' if and only if

$$[(0{,}80)F_T^0(-4.000^-) + (0{,}20)F_T^0(0^-) - F_T^0(3.000^-)]. \\ \alpha_0 \geqslant (0{,}64)\alpha_1 - (0{,}20)\alpha_2.$$

For F_T^0 uniformly distributed on [−4.000, 0], we have $F_T^0(-4.000^-) = 0$, $F_T^0(-3.000^-) = 0{,}25$, and $F_T^0(0^-) = 1$. Therefore, this inequality becomes

$$-(0{,}05)\alpha_0 \geqslant (0{,}64)\alpha_1 - (0{,}20)\alpha_2,$$

which is identical to (15). Fo $\alpha_1 = 0$ and $\alpha_0 = 1 - \alpha_2$, we obtain that A' is preferred to B' for $\alpha_2 \geqslant (1/5)$.

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