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A technique for dynamically measuring and modifying relevance while problem solving

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Abstract. Although there is general agreement that efficiency of problem resolution is strongly related to the problem representation adopted, computer problem solvers have been traditionally designed to keep the same representation throughout the whole of the problem solving process. A system able to change representation whilst the actual problem solving process occurs has advantages over traditional ones, not only because a representation change can improve the efficiency of problem resolution (as already proven through much research), but also because the choice of the most suitable representation may be decisively enhanced after learning about the problem during its resolution process. A natural and interesting way of performing representation changes is related to detecting irrelevant elements which can be removed out of the problem representation. In this paper, we deal formally with a new technique for assigning and changing the relevance of the elements involved in the representation during the problem resolution, on behalf of their respective importance so as to actually solve the problem.

Una técnica para medir y modificar dinámicamente la relevancia en la resolución de problemas

Resumen. Aunque existe un consenso general sobre la fuerte dependencia entre la eficiencia en la resolución de problemas y la representación de los mismos que se adopta, los programas resolvedores de problemas tradicionalmente se han diseñado teniendo en cuenta únicamente una sola representación a lo largo de todo el proceso de resolución. Un sistema capaz de cambiar la representación mientras el mismo proceso de resolución tiene lugar ofrece ventajas respecto de los tradicionales, no sólo porque ciertos cambios de representación pueden mejorar la eficiencia del proceso de resolución (como muchas investigaciones han demostrado), sino también debido a que la tarea de seleccionar la representación más adecuada para un problema puede facilitarse decisivamente mediante el conocimiento adquirido durante el proceso de su resolución. Una manera interesante y natural de ejecutar cambios de representación se apoya en detectar la información irrelevante que puede eliminarse de la representación del problema. En el presente trabajo, presentamos formalmente una nueva técnica para asignar y modificar valores de relevancia a los elementos que integran la representación de los problemas durante el mismo proceso de resolución, de acuerdo con la importancia que tienen para resolverlos.

1 Introduction

Problem Solvers are computational systems able to solve some problems by simulating human problem solving performance. The choice of an adequate problem representation is no doubt an important issue

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in problem solving, since it is strongly related to the efficiency of the resolution process, both in human [10, 13, 14] and computer problem solving [1, 3]. Although traditional problem solvers have not undertaken the task of changing the states representation, so that the user alone is actually responsible for choosing a suitable representation beforehand [9, 11], some efforts have been done to allow the system select an adequate representation before the actual resolution of the problem starts [2]. Other approaches, involving reformulation techniques in the context of reasoning about physical systems [1], do not imply any representation change occurring during the problem solving process.

Since these latter systems only change the representation before the resolution process starts, it becomes apparent that they allow no further alteration of the problem's states space throughout the entire resolution process. Therefore, for such systems the choice of a suitable initial representation comes to be necessarily critical. In this way, the choice of an adequate representation depends on prior knowledge the system has about the problem in question; that is to say, the more knowledge the system has about the problem, the more likely may the system choose an appropriate representation. However, it may be noted that during the resolution process, the system can gain further knowledge of the problem which may eventually happen to be crucial so as to actually enabling the system select another representation which may reduce the problem states space to a significant extent, and therefore drastically improve the efficiency of the resolution process. Besides, many psychological studies on human problem solving support the evidence that humans change continuously the conception of the problem while they are solving it, by progressively acknowledging the rising importance of some problem features, as well as disposing of those regarded as irrelevant. That is to say, humans do perform a great amount of representation changes while they are solving problems [10]. Moreover, there is a specific kind of problems, namely insight problems, which are typically solved only by means of a critical representation change occurring during the actual resolution process [8, 15].

Taking all this into consideration, in this paper we will focus on demonstrating mathematically a novel technique for measuring and modifying automatically the relevance of problem features during the resolution process. A representation change consisting in dropping those irrelevant elements from the representation not only results in a new conception of the problem, but it may also involve an outstanding improvement in the efficiency of the problem's resolution. In [7] we have already studied mathematically the relationship between a given problem and the need for performing suitable representation changes. Using this technique for measuring and modifying relevance, we have so far implemented a prototype problem solver able to perform representation changes while problem solving. This technique has been successfully tested in connection to some interesting problem examples [4, 5, 6].

In section 2, we explain our procedure of measuring a problem element's relevance. In section 3 we deal with some relevant mathematical properties about this technique. In section 4, we summarize the main conclusions of this research.

2 The measure of the relevance

A computer problem solver able to perform representation changes during the resolution of a problem requires some methods for measuring the relevance of each element in the problem representation, that is to say, the importance of each element in question, so as to actually solving the problem. Therefore the need of assigning, to every feature in the problem representation, a certain relevance level, measuring its present importance in the resolution process. Obviously, relevance levels must be dynamic ones, susceptible to be modified during the whole of the resolution process.

We have designed relevance levels so that they can oscillate inside a range of values between 0 and 100, signifying that when the relevance level of an element is 0, this element is considered as completely irrelevant, and when its relevance level reaches the value 100, this element is considered of the utmost relevance. We will define a threshold relevance value, th, under which any representation element will be considered as irrelevant. Indeed, when the relevance value associated to an element of the representation keeps on this threshold value for some period of time, the element will be considered as irrelevant and the problem solver will eliminate it from the current representation.

Now, we will study a procedure for changing the relevance levels during actual problem resolution. First of all, we show here some intuitive guidelines which have helped us design such procedure:

i) The higher the relevance level of an element in the problem representation is, the more this element will be used for the resolution of the problem. If an element in the problem representation is no longer used in connection to any task of problem resolution, it must be considered as irrelevant. On the contrary, if an element in the problem representation is frequently used, this must be considered as very relevant.

ii) The higher the relevance of an element in the problem representation is, the higher will become the importance of the tasks about problem resolution this element takes part in. An element is more relevant than another if the first one takes part in more relevant tasks than the second one.

The procedure adopted in this paper is based on the idea of considering two types of relevance levels associated to each element in the problem representation:

- **Global Relevance Level** of an element (denoted by g). It measures the importance of an element in the problem representation. It is updated periodically making use of the Recent Relevance Level associated to this element in this way:

  $$g' = f(g, r)$$

  where r stands for the recent relevance level (see next item); g stands for the global relevance level the element has when just before it is updated; f is a function $\mathbb{N} \times \mathbb{N} \to \mathbb{N}$, termed as 'Relevance change function' (see definition 1); and g' stands for the updated value of the global relevance level associated to this element.

- **Recent Relevance Level** of an element (denoted by r). It measures the importance of this element since its global relevance level was last updated. The 'Recent Relevance Level' of an element is modified in two ways:

  – When the element takes part in a task with importance t, the recent relevance level is updated in this way:

    $$r' = \max\{r, t\}$$

    where r stands for the recent relevance level which the element has just before it is updated; t stands for the importance of this element in a task; and r' stands for the updated value of the recent relevance level associated to this element.

  – The 'Recent Relevance Level' of an element is also reset to value 0 each time that its 'Global Relevance Level' is updated.

  In this way, a value 0 in the 'Recent Relevance Level' of an element involves that this element has not been used since the 'Global Relevance Level' has been last updated. When the 'Global Relevance Level' of an element is updated, it comes nearer to its present 'Recent Relevance Level' (as we will see in proposition 1). In this way, the 'Global Relevance Level' of an element will be continuously decreased when this element is not used for a long time

Now, we will define the 'relevance change function' over a set of parameters:

th (**threshold relevance value**). It must fullfil the following requirement $0 \leq th \leq 100$. This level is established for detecting irrelevant elements. Indeed, those elements whose global relevance level is under the threshold value are regarded as irrelevant and may be consequently eliminated.

k_1 must fulfill the following requirement $1 \leq k_1 \leq th$. It stands for the relevance loss derived from the lack of use of an element ($k_1 = g - f(g, 0)$)

k_2 must fulfill the following requirement $k_2 \geq 1$. It is related to the increase of the updated global relevance on behalf of the recent relevance, when the latter is greater than the global relevance function.

Definition 1 (Relevance Change Function) *The relevance change function, f is defined as follows:*

$$f(g,r) = \begin{cases} g - \text{floor}(\frac{g-r}{k_2}) & \textit{if } g \leq r \\ \max(g + \text{floor}\left(\frac{k_1 \cdot r}{g}\right) - k_1, th) & \textit{if } g > r \end{cases}$$

where floor *stands for the integer part of a real number.*

The relevance change function satisfies the following properties:

Proposition 1 *The following holds:*

i) *If $r = g$, then $f(g,r) = g$*

ii) *If $g_1 \geq g_2 \geq th$, then $f(g_1, r) \geq f(g_2, r)$*

iii) *If $r_1 \geq r_2$, then $f(g, r_1) \geq f(g, r_2)$*

iv) $\min\{g, r\} \leq f(g,r) \leq \max\{g, r\}$ *for $g \geq th$*

v) *If $g > th$ and $r < g$, then $th \leq f(g,r) < g$*

vi) *If $g > th$ and $r = 0$, then $f(g, 0) = \max\{g - k_1, th\} < g$*

vii) *If $g = th$ and $r \leq g$, then $f(g,r) = f(th, r) = th$*

viii) *If $r > g$, then $f(g,r) > g$*

ix) *If $g \geq th$, then $f(g,r) \geq th$*

x) *If $g \geq th$ and $r > th$, then $f(g,r) > th$*

PROOF.

i) $f(g,g) = g - \text{floor}((g-g)/k_2) = g$

ii) We have different cases

Case $r \leq g_2 \leq g_1$
We will take the function $h(g,r) = g + \text{floor}(k_1 \cdot r/g) - k_1$. In this case, the function $f(g,r)$ takes the value:

$$f(g,r) = \max\{h(g,r), th\}$$

First, we will prove that $h(g_2, r) \leq h(g_1, r)$

$$\begin{aligned} h(g_1,r) - h(g_2,r) &= g_1 - g_2 + \text{floor}(k_1 \cdot r/g_1) - \text{floor}(k_1 \cdot r/g_2) \\ &\geq g_1 - g_2 + \text{floor}(k_1 \cdot r/g_1 - k_1 \cdot r/g_2) \\ &= (g_1 - g_2) + \text{floor}(k_1 \cdot r \cdot (g_2 - g_1)/(g_1 \cdot g_2)) \end{aligned}$$

Since $0 < k_1 \leq th \leq g_2 \leq g_1$, and $r \leq g_2$ we have that:

$$h(g_1,r) - h(g_2,r) \geq (g_1 - g_2) + (g_2 - g_1) \geq 0$$

Therefore, $h(g_2, r) \leq h(g_1, r)$

Now, we will prove that $f(g_2, r) \leq f(g_1, r)$

If $th \leq h(g_2, r)$, then, since $th \leq h(g_2, r) \leq h(g_1, r)$ we have that

$$f(g_2, r) = \max\{h(g_2, r), th\} = h(g_2, r)$$

Therefore, we have that $f(g_2, r) = h(g_2, r) \leq h(g_1, r) = f(g_1, r)$

If $th > h(g_2, r)$ then

$$f(g_2, r) = \max\{h(g_2, r), th\} = th \leq f(g_1, r)$$

Case $g_2 \leq g_1 \leq r$

$$\begin{aligned} f(g_1, r) - f(g_2, r) &= g_1 - \text{floor}((g_1 - r)/k_2) - g_2 + \text{floor}((g_2 - r)/k_2) \\ &= (g_1 - g_2) + \text{floor}((g_2 - r)/k_2) - \text{floor}((g_1 - r)/k_2) \\ &\geq (g_1 - g_2) + \text{floor}((g_2 - r)/k_2 - (g_1 - r)/k_2) \\ &= (g_1 - g_2) + \text{floor}(-(g_1 - g_2)/k_2) \\ &\geq (g_1 - g_2) \cdot (1 + \text{floor}(-1/k_2)) \end{aligned}$$

Since $k_2 \geq 1$, we have that $\text{floor}(-1/k_2) = -1$.

Consequently, $f(g_1, r) - f(g_2, r) \geq 0$.

Therefore,

$$f(g_2, r) \leq f(g_1, r)$$

Case $g_2 \leq r \leq g_1$

Since $g_2 \leq r \leq r$, by the first case, we have that $f(g_2, r) \leq f(r, r)$.

Since $r \leq r \leq g_1$, by the second case, we have that $f(r, r) \leq f(g_1, r)$.

Therefore, we have

$$f(g_2, r) \leq f(g_1, r).$$

iii) We have different cases:

Case $r_2 \leq r_1 \leq g$

$$\begin{aligned} f(g, r_1) - f(g, r_2) &= \text{floor}(k_1 \cdot r_1/g) - \text{floor}(k_1 \cdot r_2/g) \\ &\geq \text{floor}(k_1 \cdot r_1/g - k_1 \cdot r_2/g) \\ &\geq \text{floor}(k_1 \cdot (r_1 - r_2)/g) \end{aligned}$$

Since $k_1 > 0$, $(r_1 - r_2) > 0$ and $g > 0$, we have that $f(g, r_1) - f(g, r_2) \geq 0$.

Therefore,

$$f(g, r_1) \geq f(g, r_2).$$

Case $g \leq r_2 \leq r_1$

$$\begin{aligned} f(r_1, g) - f(r_2, g) &= \text{floor}((g - r_2)/k_2) - \text{floor}((g - r_1)/k_2) \\ &\geq \text{floor}((g - r_2)/k_2 - (g - r_1)/k_2) \\ &\geq \text{floor}((r_1 - r_2)/k_2) \end{aligned}$$

Since $k_2 \geq 1$ and $(r_1 - r_2) > 0$ we have that $f(g, r_1) - f(g, r_2) \geq 0$.

Therefore,

$$f(g, r_1) \geq f(g, r_2).$$

Case $r_2 \leq g \leq r_1$

Since $r_2 \leq g \leq g$, by the first case, we have $f(g, r_2) \leq f(g, g)$.
Since $g \leq g \leq r_1$, by the second case, we have $f(g, g) \leq f(g, r_1)$.
Therefore,

$$f(g, r_2) \leq f(g_1, r_1).$$

iv) By properties iii) and i), we have that

$$\min\{g, r\} = f(\min\{g, r\}, \min\{g, r\}) \leq f(g, r) \leq f(\max\{g, r\}, \max\{g, r\}) = \max\{g, r\}$$

v) Since $k_1 \geq 1$, $r \leq g - 1$, and $g > 0$, we have that:

$$\text{floor}(k_1 \cdot r/g) \leq \text{floor}(k_1 \cdot (g-1)/g) = k_1 + \text{floor}(-1/g) \leq k_1 - 1 < k_1.$$

Therefore:

$$f(g, r) = \max\{g + \text{floor}(k_1 \cdot r/g) - k_1, th\} \leq \max\{g + k_1 - 1 - k_1, th\} = \max\{g - 1, th\}.$$

As $g - 1 < g$ and $th < g$, we have that

$$th \leq f(g, r) < g.$$

vi) If $r = 0$, then $f(g, r) = f(g, 0) = \max\{g + \text{floor}(k_1 \cdot 0/g) - k_1, th\} = \max\{g - k_1, th\}$

vii) If $r = th$, then $f(g, r) = f(th, th) = th$
If $r < th$, then, since $1 \leq k_1 \leq th$ and $r \leq th - 1$, we have that:

$$\begin{aligned} th + \text{floor}(k1 \cdot r/\,th) - k_1 &\leq th + \text{floor}(k_1 \cdot (th - 1)/\,th) - k_1 \\ &\leq th + \text{floor}(-k1/\,th) \\ &= th - 1 \\ &< th \end{aligned}$$

Therefore,

$$f(th, r) = \max\{th + \text{floor}(k_1 \cdot r/\,th) - k_1, th\} = th\,.$$

viii) If $r > g$, then, since $(g - r)/k_2 < 0$, if $r > g$, we have that $\text{floor}((g - r)/k_2) \leq -1$.
Therefore, if $r > g$, then

$$f(g, r) = g - \text{floor}((g - r)/k_2) \geq g + 1 > g.$$

ix) We show this statement considering different cases:
If $r > g$, then, by viii), we have that $f(g, r) > g \geq th$
If $r = g$, then, by i), we have that $f(g, r) = g \geq th$
If $r < g$ and $g > th$ then, by v), we have that $f(g, r) \geq th$
If $r < g$ and $g = th$ then, by vii), we have that $f(g, r) = th \geq th$

x) We show this statement considering different cases:
If $g = th$, by viii) we have that $f(g, r) > g = th$,
If $g > th$, by iv), we have that $f(g, r) \geq \min\{g, r\} > th$.

■

3 Relation between use and global relevance level

In this section we will undertake a formal study of the relation between the use made of an element and its global relevance level, by analyzing the evolution of the global relevance in elements that are used regularly. We will consider the successions g_n, r_n indicating respectively the global and recent relevance levels of a element in the instant n (n indicates exactly the number of global relevance updates which have been carried out). We will consider that this element is used regularly; that is to say, that it always takes part in a task with a relevance R being performed periodically each N updates of global relevance and the rest of time is not used. In this way,

$$r_n = \begin{cases} R & \text{if } n \bmod (N+1) = 0 \\ 0 & \text{if } n \bmod (N+1) \neq 0 \end{cases}$$

The succession g_n is defined through the succession r_n:

$$\forall n > 0 \qquad g_n = f(g_{n-1}, r_{n-1})$$

By the next theorems and propositions we will show the following statements:

i) Any initial values in the Global Relevance, g_0, however distant, will in the end approach to similar levels.

ii) After a certain update, the global relevance g_n will constantly fluctuate between some values, and these values will be under R.

iii) The greater R and the lesser N are, the greater will the global relevance level be.

iv) The greater N is, the greater will fluctuations in the global relevance be.

The following proposition states that the global relevance level is always greater than the threshold relevance level, th.

Proposition 2 *If* $g_0 \geq th$*, then* $\forall n \in \mathbb{N} \;\; g_n \geq th$.

PROOF. It is proven by induction.

Base case $n = 0$, $g_0 \geq th$

Inductive case Suppose that $g_n \geq th$. By ix) of proposition 1, we have that $g_{n+1} = f(g_n, r) \geq th$. ■

The following theorem is aimed to define recursively the succession g_n:

Theorem 1 *We have that:*

i) $\forall n \in \mathbb{N} \quad g_{(n+1)\cdot(N+1)} = \max\{g_{n\cdot(N+1)+1} - k_1 \cdot N, th\}$

ii) $\forall m \geq 1 \quad g_{m\cdot(N+1)} = \max\{f(g_{(m-1)\cdot(N+1)}, R) - k_1 \cdot N, th\}$

iii) $\forall n \in \mathbb{N} \quad \forall i \in \{1, \ldots, N+1\}$, *if* $g_{n\cdot(N+1)} \leq R$ *then*

$$g_{n\cdot(N+1)+i} = \max\{g_{n\cdot(N+1)} - \text{floor}((g_{n\cdot(N+1)} - R)/k_2) - k_1 \cdot (i-1), th\}$$

iv) $\forall n \in \mathbb{N} \quad \forall i \in \{1, \ldots, N+1\}$, *if* $g_{n\cdot(N+1)} > R$ *then*

$$g_{n\cdot(N+1)+i} = \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_n \cdot (N+1)) - k_1 \cdot i, th\}$$

PROOF.

i) In case that $N = 0$, we have that $g_{n+1} = \max\{g_{n+1}, th\} = g_{n+1}$.
We will consider the case that $N > 0$.
Let $n \in \mathbb{N}$ and let $i \in \{1, \ldots, N\}$.
We have that $g_{n\cdot(N+1)+i+1} = f(g_{n\cdot(N+1)+i}, r_{n\cdot(N+1)+i})$.
Since $n \cdot (N+1) + i \bmod (N+1) = i \neq 0$, we have that:

$$g_{n\cdot(N+1)+i+1} = f(g_{n\cdot(N+1)+i}, 0) = \max\{g_{n\cdot(N+1)+i} - k_1, th\}.$$

By applying recursively this expression i times, we have that

$$g_{n\cdot(N+1)+i+1} = \max\{g_{n\cdot(N+1)+1} - k_1 \cdot i, th\}.$$

Specifically, for $i = N$, we have that

$$g_{n\cdot(N+1)+N+1} = g_{(n+1)\cdot(N+1)} = \max\{g_{n\cdot(N+1)+1} - k_1 \cdot N, th\}.$$

ii) Let $n = m - 1$ and we will study the value of $g_{n\cdot(N+1)+1}$. We have these two cases:

Case $g_{n\cdot(N+1)} < R$

$$g_{n\cdot(N+1)+1} = g_{n\cdot(N+1)} - \text{floor}((g_{n\cdot(N+1)} - R)/k_2).$$

By i) we have that:

$$g_{(n+1)\cdot(N+1)} = \max\{g_{n\cdot(N+1)} - \text{floor}((g_{n\cdot(N+1)} - R)/k_2) - k_1 \cdot N, th\}.$$

Case $g_{n\cdot(N+1)} \geq R$

$$g_{n\cdot(N+1)+1} = \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_{n\cdot(N+1)}) - k_1, th\}.$$

By i), we have that:

$$g_{(n+1)\cdot(N+1)} = \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_{n\cdot(N+1)}) - k_1 \cdot (N+1), th\}.$$

In any of these two cases, we have that

$$g_{(n+1)\cdot(N+1)} = \max\{f(g_{n\cdot(N+1)}, R) - k_1 \cdot N, th\}$$

iii) We will prove it by induction

Base case $i = 1$
Since $r_{n\cdot(N+1)} = R$, we have that $g_{n\cdot(N+1)+1} = f(g_{n\cdot(N+1)}, R) \geq th$.
Besides, we have that:

$$\begin{aligned} g_{n\cdot(N+1)+1} &= f(g_{n\cdot(N+1)}, R) = g_{n\cdot(N+1)} - \text{floor}((g_{n\cdot(N+1)} - R)/k_2) \\ &= g_{n\cdot(N+1)} - \text{floor}((g_{n\cdot(N+1)} - R)/k_2) - k_1 \cdot (1-1). \end{aligned}$$

Inductive case We will suppose that the following holds:
$g_{n\cdot(N+1)+i} = \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_{n\cdot(N+1)}) - k_1 \cdot i, th\}$ where $1 \leq i < N+1$.
Since $r_{n\cdot(N+1)+i} = 0$, we have that:

$$\begin{aligned} g_{n\cdot(N+1)+i+1} &= f(g_{n\cdot(N+1)+i}, 0) \\ &= \max\{g_{n\cdot(N+1)+i} - k_1, th\} \\ &= \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_{n\cdot(N+1)}) - k_1 \cdot (i+1), th\} \end{aligned}$$

iv) Let $n \in \mathbb{N}$ and let $i \in \{1, \ldots, N+1\}$
We will prove it by induction

Base case

Since $r_{n\cdot(N+1)} = R$, we have that

$$g_{n\cdot(N+1)+1} = f(g_{n\cdot(N+1)}, R) = \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_{n\cdot(N+1)}) - k_1, th\}$$

Inductive case

$g_{n\cdot(N+1)+i} = \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_{n\cdot(N+1)}) - k_1 \cdot i, th\}$ and $1 \leq i < N+1$.
Since $r_{n\cdot(N+1)+i} = 0$

$$\begin{aligned} g_{n\cdot(N+1)+i+1} &= f(g_{n\cdot(N+1)+i}, 0) \\ &= \max\{g_{n\cdot(N+1)+i} - k_1, th\} \\ &= \max\{g_{n\cdot(N+1)} + \text{floor}(k_1 \cdot R/g_{n\cdot(N+1)}) - k_1 \cdot (i+1), th\} \end{aligned}$$

■

Next, we will prove that when n tends to infinite, g_n fluctuates within a constant range. In order to prove this, we will study the evolution of g_n in certain specific moments. First, we will define the succession, p_n, as follows:

$$p_n = g_{n\cdot(N+1)}.$$

Next, we will show that p_n tends to a fixed point. By ii) in theorem 1, we may define p_n recursively as follows:

$$p_0 = g_0$$
$$p_{n+1} = \max\{f(p_n, R) - k_1 \cdot N, th\}$$

In lemmas 1 and 2 we will show a relation between p_n and p_{n+1}. Both these lemmas are used in theorems 2 and 3 for showing that the succession p_n tends to a fixed value.

Lemma 1 *The following holds:*

i) *If* $th \leq p_n \leq R-1$, *then* $p_{n+1} = R$.

ii) *If* $p_n = R$, *then* $p_{n+1} = R$.

iii) *If* $p_n \geq R+1$, *then* $R \leq p_{n+1} < p_n$.

PROOF.

i) We have that:

$$p_{n+1} = \max\{p_n - \text{floor}((p_n - R)/k_2), th\} \geq \max\{p_n - \text{floor}((R-1-R)/k_2), th\}$$

Since $k_2 \geq 1$, we have that $\text{floor}(-1/k_2) = -1$, and therefore

$$p_{n+1} \geq \max\{p_n - \text{floor}(-1/k_2), th\} = \max\{p_n + 1, th\}$$

Since $p_n \geq th$, we have that $p_n + 1 > th$, and therefore:

$$p_{n+1} \geq \max\{p_n + 1, th\} = p_n + 1 > p_n$$

Besides, we have that $p_{n+1} \leq R$. Therefore

$$p_{n+1} = \max\{f(p_n, R), th\} \leq \max\{f(R, R), R\} = R$$

ii) We have that:

$$p_{n+1} = \max\{R + \text{floor}(k_1 \cdot R/R) - k_1, th\} = \max\{R + k_1 - k_1, th\} = \max\{R, th\}.$$

Since $R > th$, we have that

$$p_{n+1} = R$$

iii) We have that:

$$\begin{aligned} p_{n+1} &= \max\{p_n + \text{floor}(k_1 \cdot R/p_n) - k_1, th\} \\ &\leq \max\{p_n + \text{floor}(k_1 \cdot (p_n - 1)/p_n) - k_1, th\} \\ &\leq \max\{p_n + k_1 + \text{floor}(-k_1/p_n) - k_1, th\} \\ &\leq \max\{p_n + \text{floor}(-k1/pn), th\}. \end{aligned}$$

Since $k_1 \leq th \leq p_n$ and $\text{floor}(-k_1/p_n) = -1$, we have that

$$p_{n+1} \leq \max\{p_n - 1, th\}.$$

Since $th < R$ and $R < p_n$, we have that $th \leq p_n - 1$ and therefore

$$p_{n+1} \leq \max\{p_n - 1, th\} = p_n - 1 < p_n.$$

Besides, we have that $p_{n+1} \geq R$

$$p_{n+1} = \max\{f(p_n, R), th\} \geq \max\{f(R, R), th\} = R.$$

■

Theorem 2 *If $N = 0$, then we have that:*

$$\exists n \in \mathbb{N} \quad \forall m \geq n \qquad p_m = R$$

PROOF. We will prove that $\exists n \geq 0$ such that $p_n = R$. Once we have proved it, we will prove this lemma by applying ii)in lemma 1. We will consider these cases:

Case $th < p_0 \leq R - 1$
By i) in lemma 1, we have that $p_1 = R$. By ii) in lemma 1, we have that

$$\forall m \geq 1 \qquad p_m = R$$

Case $p_0 = R$
By ii) in lemma 1, we have that

$$\forall m \geq 0 \qquad p_m = R$$

Case $p_0 \geq R + 1$
By iii) in lemma 1 we have that:

$$\exists n \geq 0 \text{ such that} \quad p_n = R$$

Therefore, by ii) in lemma 1, we have that

$$\forall m \geq n \qquad p_m = R$$

■

Lemma 2 *If $N \geq 1$, then we have that:*

i) *If $p_n < \max\{R - k_1 \cdot k_2 \cdot N - 1, th\}$ then*

$$p_n < p_{n+1} \leq \max\{R - k_1 \cdot k_2 \cdot N, th\}$$

ii) *If $p_n > \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}$ and $th \leq p_n \leq R$, then*

$$\max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\} \leq p_{n+1} < p_n$$

iii) *If $p_n > \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}$ and $p_n \geq R + 1$, then*

$$\max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\} \leq p_{n+1} < p_n$$

iv) *If $p_n = \max\{R - k_1 \cdot k_2 \cdot N + i, th\}$ where $0 \leq i \leq k_2 - 1$, then*

$$p_{n+1} = p_n$$

PROOF.

i) Since $p_n \leq \max\{R - k_1 \cdot k_2 \cdot N - 1, th\}$, we have that $p_n \leq R - k_1 \cdot k_2 \cdot N - 1 < R$

We will prove that $p_{n+1} > p_n$.

Since we have that $p_n < R$.

$$\begin{aligned} p_{n+1} &= \max\{f(p_n, R) - k_1 \cdot N, th\} \\ &= \max\{p_n - \text{floor}((p_n - R)/k_2) - k_1 \cdot N, th\} \\ &\geq \max\{p_n - \text{floor}((R - k_1 \cdot k_2 \cdot N - 1 - R)/k_2) - k_1 \cdot N, th\} \\ &\geq \max\{pn + k_1 \cdot N - \text{floor}(-1/k2) - k_1 \cdot N, th\} \\ &= \max\{p_n - \text{floor}(-1/k_2), th\}. \end{aligned}$$

Since $k_2 \geq 1$, we have that $\text{floor}(-1/k_2) = -1$. Therefore

$$p_{n+1} \geq \max\{p_n - \text{floor}(-1/k_2), th\} = \max\{p_n + 1, th\}.$$

Since $p_n \geq th$, we have that $p_n + 1 > th$. Therefore: $p_{n+1} \geq \max\{p_n + 1, th\} = p_n + 1 > p_n$.

We will prove that $p_{n+1} \leq \max\{R - k_1 \cdot k_2 \cdot N, th\}$.

$$p_{n+1} = \max\{f(p_n, R) - k_1 \cdot N, th\} \leq \max\{f(R - k_1 \cdot k_2 \cdot N - 1, R) - k_1 \cdot N, th\}$$

Since $R - k_1 \cdot k_2 \cdot N - 1 < R$, we have that

$$\begin{aligned} p_{n+1} &\leq \max\{R - k_1 \cdot k_2 \cdot N - 1 - \text{floor}((R - k_1 \cdot k_2 \cdot N - 1 - R)/k_2) - k_1 \cdot N, th\} \\ &\leq \max\{R - k_1 \cdot k_2 \cdot N - 1 + k_1 \cdot N + 1 - k_1 \cdot N, th\} \\ &= \max\{R - k_1 \cdot k_2 \cdot N, th\} \end{aligned}$$

ii) We will prove that $p_{n+1} < p_n$.

Since $p_n \leq R$, we have that:

$$p_{n+1} = \max\{f(p_n, R) - k_1 \cdot N, th\} = \max\{p_n - \text{floor}((p_n - R)/k_2) - k_1 \cdot N, th\}.$$

Since $p_n \geq R - k_1 \cdot k_2 \cdot N + k_2$, we have that:

$$p_{n+1} \leq \max\{p_n - \text{floor}((R - k_1 \cdot k_2 \cdot N + k_2 - R)/k_2) - k_1 \cdot N, th\} = \max\{p_n - 1, th\}.$$

Since $p_n > th$, we have that $th \leq p_n - 1$. Therefore:

$$p_{n+1} \leq \max\{p_n - 1, th\} = p_n - 1 < p_n$$

We will prove that $p_{n+1} \geq \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}$
Since $R - k_1 \cdot k_2 \cdot N + k_2 \leq p_n \leq R$, we have that

$$\begin{aligned} p_{n+1} &= \max\{f(p_n, R) - k_1 \cdot N, th\} \\ &\geq \max\{f(R - k_1 \cdot k_2 \cdot N + k_2, R) - k_1 \cdot N, th\} \\ &\geq \max\{R - k_1 \cdot k_2 \cdot N + k_2 - \text{floor}((R - k_1 \cdot k_2 \cdot N + k_2 - R)/k_2) - k_1 \cdot N, th\} \\ &\geq \max\{R - k_1 \cdot k_2 \cdot N + k_2 + k_1 \cdot N - 1 -_1 \cdot N, th\} \\ &= \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\} \end{aligned}$$

iii) We will prove that $p_{n+1} < p_n$.
Since $p_n > R$, we have that:

$$\begin{aligned} p_{n+1} &= \max\{f(p_n, R) - k_1 \cdot N, th\} \\ &= \max\{p_n + \text{floor}(k_1 \cdot R/p_n) - k_1 \cdot (N+1), th\} \\ &\leq \max\{p_n + \text{floor}(k_1 \cdot p_n/p_n) - k_1 \cdot (N+1), th\} \\ &= \max\{p_n - k_1 \cdot N, th\} \leq \max\{p_n - 1, th\}. \end{aligned}$$

Since $th < R + 1 \leq p_n$, we have that $th \leq p_n - 1$

$$p_{n+1} \leq \max\{p_n - 1, th\} = p_n - 1 < p_n$$

We will prove that $p_{n+1} \geq \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}$.
Since $p_n \geq R - k_1 \cdot k_2 \cdot N + k_2$, we have that:

$$\begin{aligned} p_{n+1} &= \max\{f(p_n, R) - k_1 \cdot N, th\} \\ &= \max\{p_n + \text{floor}(k_1 \cdot R/p_n) - k_1 \cdot (N+1), th\} \\ &\geq \max\{f(R - k_1 \cdot k_2 \cdot N + k_2, R) - k_1 \cdot N, th\} \end{aligned}$$

Since $k_1 \geq 1$ and $N \geq 1$, we have that $R - k_1 \cdot k_2 \cdot N + k_2 \leq R - k_2 + k_2 = R$

$$\begin{aligned} p_{n+1} &\geq \max\{R - k_1 \cdot k_2 \cdot N + k_2 - \text{floor}((R - k_1 \cdot k_2 \cdot N + k_2 - R)/k_2) - k_1 \cdot N, th\} \\ &\geq \max\{R - k_1 \cdot k_2 \cdot N + k_2 + k_1 \cdot N - 1 - k_1 \cdot N, th\} \\ &= \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}. \end{aligned}$$

iv) We will consider these two cases:

Case $p_n = R - k_1 \cdot k_2 \cdot N + i \geq th$
Since $p_n \leq R - k_1 \cdot k_2 \cdot N + k_2 - 1 \leq R - k_2 + k_2 - 1 = R - 1 < R$, we have that:

$$\begin{aligned} p_{n+1} &= \max\{p_n - \text{floor}((p_n - R)/k_2) - k_1 \cdot N, th\} \\ &= \max\{p_n - \text{floor}((R - k_1 \cdot k_2 \cdot N + i - R)/k_2) - k_1 \cdot N, th\} \\ &= \max\{p_n - \text{floor}(i/k_2), th\} \end{aligned}$$

Since $0 \leq i \leq k_2 - 1$, we have that $\text{floor}(i/k_2) = 0$, and consequently:

$$p_{n+1} = \max\{p_n - \text{floor}((i/k_2), th\} = \max\{p_n, th\} = p_n$$

Case $p_n = th \geq R - k_1 \cdot k_2 \cdot N + i$
We have that:

$$\begin{aligned} th - \text{floor}((th - R)/k_2) - k_1 \cdot N &\leq th - \text{floor}((R - k_1 \cdot k_2 \cdot N + i - R)/k_2) - k_1 \cdot N \\ &\leq th + k_1 \cdot N - \text{floor}(i/k_2) - k_1 \cdot N \\ &= th \end{aligned}$$

Therefore,

$$p_{n+1} = \max\{th - \text{floor}((th - R)/k_2) - k_1 \cdot N, th\} = th = p_n.$$

■

Theorem 3 *If* $N \geq 1$*, we have that:*

i) *If* $p_0 \leq \max\{R - k_1 \cdot k_2 \cdot N, th\}$*, then* $\exists n \geq 0 \;\; \forall m \geq n \quad p_m = \max\{R - k_1 \cdot k_2 \cdot N, th\}$.

ii) *If* $p_0 \geq \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}$*, then* $\exists n \geq 0 \;\; \forall m \geq n \quad p_m = \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}$

iii) *If* $\max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\} \leq p_0 \leq \max\{R - k_1 \cdot k_2 \cdot N, th\}$*, then* $\forall m \geq 0 \quad p_m = p_0$.

PROOF.

i) By taking into account i) in lemma 2, $\exists n > 0 \quad p_n = \max\{R - k_1 \cdot k_2 \cdot N, th\}$.
By taking into account iv) in lemma 2, $\forall m > n \quad p_m = p_n$.

ii) By taking into account ii) in lemma 2, $\exists n > 0 \quad p_n = \max\{R - k_1 \cdot k_2 \cdot N + k_2 - 1, th\}$.
By taking into account iv) in lemma 2, $\forall m > n \quad p_m = p_n$.

iii) By taking into account iv) in lemma 2, $\forall m \geq 0 \quad p_m = p_0$.

■

4 Conclusions

As has been seen, assigning relevance values to the various elements shaping a problem representation is really useful in order to solve it. This paper describes some new techniques for establishing and modifying such relevance values within the performance of a computer problem solver we have implemented and which, unlike its predecessors, is able to execute crucial changes in the problem representation in virtually any moment of the solving process. These representation changes occur as an autonomous fulfillment of our problem solver, with no need for the user to order or activate them, being related to the measure of relevance values in the way described in this paper. Thus, automatic representation changes within the system, dependent on relevance values of the problem elements, offer a valid simulation of unconscious human behavior while problem solving. Further specific details about the architecture and performance of our problem solver are to be found in some of the references below.

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